50 mins MAX

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5th Year Honours Maths Test on Algebra 1(a.)

Name: M. Selkirk

1.) Solve the equation $x^2 - 2\sqrt{3}x - 9 = 0$, giving your answers in the form $a\sqrt{3}$, where $a \in \mathbb{Q}$.

$$\frac{2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(1)(-9)}}{2(1)} = 3\sqrt{3}, \ -\sqrt{3}.$$

(0, 2, 5, 7, 10)

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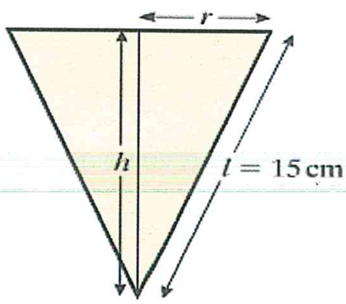
2.) A cone has a radius r cm and a vertical height h cm. If the slant height $l = 15$ cm, and using Pythagoras' theorem, Ex 1·6 Q11 p 27

(i) express h in terms of r.

(ii) Hence find the value of h when $r = 5$ cm.

(iii) At what value of h will the radius r be equal to half the slant height l? Give your answer correct to the nearest cm.



(i)

$h^2 + r^2 = l^2$ $\quad h = \sqrt{l^2 - r^2} = \sqrt{225 - r^2}$

(2, 5) 5

(ii)

$h = \sqrt{225 - (5)^2} = \sqrt{200} = 14.42$ or $10\sqrt{2}$

(2, 5) 5

(iii)

$r = 7.5$ $\quad h = \sqrt{225 - (7.5)^2} = 13$

(2, 5) 5

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3.) If $x^2 - px + 1$ is a factor of $ax^3 + bx + c$, prove that

section 1.5

(i) $c = ap$ (ii) $c^2 = a(a - b)$

(i)

$$\begin{array}{r} ax + ap \\ x^2 - px + 1 \overline{)\, ax^3 + 0x + bx + c} \\ ax^3 - apx^2 + ax \\ \hline apx^2 + (b-a)x + c \\ apx^2 - ap^2x + ap \\ \hline \end{array}$$

(0, 2, 5, 7, 10)

$c = ap$ ✓

(ii)

$(b - a) = -ap^2$

$p = \frac{c}{a}$ $\quad b - a = -\frac{\cancel{a}c^2}{a^{\cancel{2}}}$

(0, 2, 5)

$b - a = -\frac{c^2}{a}$ $\quad a(b - a) = -c^2 \quad (\times -1)$

$a(a - b) = c^2$ ✓

4.) Write c in terms of the other variables in each of the following.

(i) $d = \sqrt{\frac{a - b}{ac}}$ (ii) $b = \frac{2c - 1}{c - 1}$ Ex 1.6 Q10

(i)

$d^2 = \frac{a - b}{ac}$

(0, 2, 5, 7, 10)

$acd^2 = a - b$

$c = \frac{a - b}{ad^2}$ ✓

(ii)

$b = \frac{2c - 1}{c - 1}$

$b(c - 1) = 2c - 1$

$bc - b = 2c - 1$

$bc - 2c = b - 1$

$c(b - 2) = b - 1$

$c = \frac{b - 1}{b - 2}$ ✓

(0, 2, 5, 7, 10)

5.) Factorise the following: (i) $y^3 - 1$ (ii) $1000a^3 - 343b^3$

(i) $y^3 - 1 = (y - 1)(y^2 + y + 1)$ ✓ (0, 2, 5)

(ii) $1000a^3 - 343b^3 = (10a)^3 - (7b)^3$

(0, 2, 5, 7, 10)

$= (10a - 7b)((10a)^2 + (10a)(7b) + (7b)^2)$ ✓

$= (10a - 7b)(100a^2 + 70ab + 49b^2)$

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6.) Factorise the following:

(i) $51 + x^2 + 20x$

$(x+3)(x+17)$ ✓

(0, 2, 5)

(ii) $-x^2 + 169$

$169 - x^2$ ✓

$(13-x)(13+x)$

(0, 2, 5)

(iii) $ax + by + ay + bx$

$a(x+y) + b(x+y)$

$(a+b)(x+y)$ ✓

(0, 2, 5)

(iv) $a^2 + b^2 - 2ab$

$(a-b)(a-b)$ ✓

(0, 2, 5)

(v) $a^2 + b^2 - c^2 - 2ab$

$= a^2+b^2-2ab-c^2$

$= (a-b)^2 - c^2$

$= (a-b-c)(a-b+c)$ ✓

(0, 2, 5, 7, 10)

7.) Simplify: (i) $\dfrac{8x^3 + 27}{4x^2 - 9}$

$\dfrac{(2x)^3 + 3^3}{(2x)^2 - 3^2} = \dfrac{(2x+3)(4x^2-6x+9)}{(2x+3)(2x-3)}$

$= \dfrac{4x^2-6x+9}{2x-3}$ ✓ (0, 2, 5, 7, 10)

(ii) $\dfrac{3t - 10}{3t - 2} - \dfrac{8}{2 - 3t}$

$\dfrac{3t-10}{3t-2} + \dfrac{8}{3t-2} = \dfrac{3t-2}{3t-2} = 1$ ✓

(0, 2, 5, 7, 10)

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(iii) $\dfrac{2x}{x + 3} + \dfrac{3x}{x - 3} - \dfrac{5x^2 + 9}{x^2 - 9}$

$\dfrac{(x-3)\,2x}{(x-3)\,x+3} + \dfrac{(x+3)\,3x}{(x+3)\,x-3} - \dfrac{5x^2+9}{(x-3)(x+3)}$

$\dfrac{(x-3)(2x) + (x+3)3x - (5x^2+9)}{(x-3)(x+3)}$

(0, 2, 5, 7, 10)

$\dfrac{\cancel{2x^2} - \cancel{6x} + \cancel{3x^2} + 9x - \cancel{5x^2} - 9}{(x-3)(x+3)}$

$\dfrac{3x-9}{(x-3)(x+3)} = \dfrac{3(x-3)}{(x-3)(x+3)} = \dfrac{3}{x+3}$ ✓

8.)

(i) The area of a rectangle can be expressed as $2x^2 + 3x - 20$. The length of the rectangle is $x + 4$. Find the breadth of the rectangle in terms of x:

$b = \dfrac{2x^2+3x-20}{x+4} = \dfrac{(2x-5)(x\cancel{+4})}{(x\cancel{+4})} = 2x-5$

(0, 2, 5)

(ii) This rectangle is used as a base for a rectangular box. The volume of the box can be expressed as $6x^3 + 7x^2 - 63x + 20$. Find the height of the box in terms of x:

$$
\begin{array}{r|l}
 & 3x - 1 \\
2x^2+3x-20 & 6x^3 + 7x^2 - 63x + 20 \\
 & 6x^3 + 9x^2 - 60x \\
\hline
 & -2x^2 - 3x + 20 \\
 & -2x^2 - 3x + 20 \\
\hline
 & 0
\end{array}
$$

✓

Ans : $3x-1$

(0, 2, 5, 7, 10)

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9.) The future value of €P, invested for 3 years at i %, is given by the formula :

$$A = P\left(1 + \frac{i}{100}\right)^3$$

Ex 1·6 Q9

(i) Find, i in terms of P and A.

$$\frac{A}{P} = \left(1 + \frac{i}{100}\right)^3$$

$$\sqrt[3]{\frac{A}{P}} = 1 + \frac{i}{100} \Rightarrow \sqrt[3]{\frac{A}{P}} - 1 = \frac{i}{100}$$

$$100\left(\sqrt[3]{\frac{A}{P}} - 1\right) = i \quad ✓$$

(0, 2, 5, 7, 10)

(ii) If €2500 invested 3 years ago has a present value of €2650, find the rate of interest correct to one decimal place:

Ex 1·6 Q9

$$i = 100\left(\sqrt[3]{\frac{2650}{2500}} - 1\right)$$

$$i = 1{\cdot}96\%.$$

$$= 2\% \quad ✓$$

(0, 2, 5, 7, 10)

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