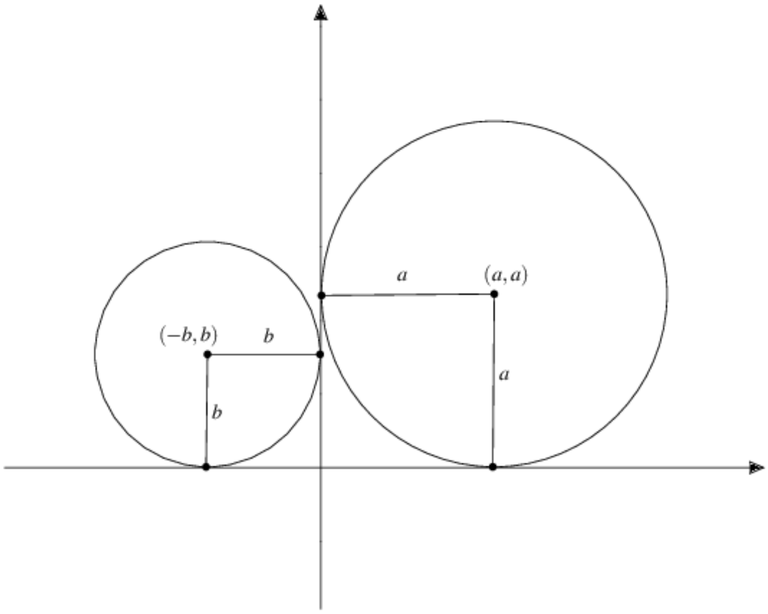
Question 1

| Q2 | Model Solution – 25 Marks | Marking Notes |
|---|---|---|
| (a) | $y - 6 = \frac{1}{7}(x + 1)$<br>$x - 7y + 43 = 0$ | Scale 10C (0, 3, 7, 10)<br><br>*Low Partial Credit:*<br>• equation of line formula with some relevant substitution<br><br>*High Partial Credit:*<br>• equation of line not in required form |
| (b) | $D = \frac{\lvert ax_1 + by_1 + c\rvert}{\sqrt{a^2 + b^2}}$<br>$D = \frac{\lvert 3(-g) + 4(-f) - 21\rvert}{\sqrt{3^2 + 4^2}}$<br>$25 = \lvert -3g - 4f - 21\rvert$<br>$-3g - 4f - 21 = \pm 25$<br>$\Rightarrow 3g + 4f = -46 \quad \ldots \text{(i)}$<br>$\text{and } 3g + 4f = 4 \quad \ldots \quad \text{(ii)}$<br><br>But $(-g, -f) \in x - 7y + 43 = 0$<br>$\Rightarrow -g + 7f + 43 = 0 \ \ldots \ \text{(iii)}$<br>$\Rightarrow g = 7f + 43$<br><br>Solving : $g = 7f + 43$ and $3g + 4f = -46$<br>$f = -7 \text{ and } g = -6$<br>Centre (6, 7)<br>$(x-6)^2 + (y-7)^2 = 25$<br><br>**or**<br>Solving: $g = 7f + 43$ and $3g + 4f = 4$<br>$f = -5 \text{ and } g = 8$<br>Centre (- 8, 5)<br><br>$(x+8)^2 + (y-5)^2 = 25$ | Scale 15D (0, 4, 7,11,1 5)<br>*Low Partial Credit*<br>• some correct substitution into relevant formula (line, circle, perpendicular distance).<br><br>*Mid Partial Credit*<br>• one relevant equation in *g* and *f*<br>• ( either(i) or (ii) or (iii))<br><br>*High Partial Credit*<br>• two relevant equations ( either (i) and (iii) or (ii) and (iii)) |

Question 2

Consider the diagram below:




We can see from this diagram that if $(x,y)$ is the centre of a circle that has both the $x$-axis and the $y$-axis as tangents, then either

- Case 1: $y=x$
- Case 2: $y=-x$

In either case the radius $r$ is $\pm x$. Since the radius is positive, we must have $r=|x|$.
Case 1: $y=x$. We are also told that $x+2y-6=0$. Subsituting $x$ for $y$ in the latter equation gives

$$x+2x-6=0 \Leftrightarrow 3x-6=0 \Leftrightarrow x=2.$$

Now $y=x$ so $y=2$. Therefore the centre of the circle has co-ordinates $(2,2)$ and the radius is 2. Therefore in this case the circle has equation

$$(x-2)^2+(y-2)^2=4.$$

Case 2: $y=-x$. As before we use this to substitute $-x$ for $y$ in the equation $x+2y-6=0$. This gives

$$x+2(-x)-6=0 \Leftrightarrow -x-6=0 \Leftrightarrow x=-6.$$

It follows that $y=-(-6)=6$. So in this case the centre has co-ordinates $(-6,6)$ and the radius is 6. So this circle has equation

$$(x+6)^2+(y-6)^2=36.$$

MODEL ANSWER BY themathstutor.ie ONLINE SUPPORT SYSTEM FOR PROJECT MATHS

**Question 4** **(25 marks)**

**(a)** Write down the equation of the circle with centre $(-3,2)$ and radius 4.

Let the centre of the circle $(h,k) = (-3,2)$ and $r = 4$. So the equation of the circle is

$$\begin{aligned} (x-h)^2+(y-k)^2 &= r^2 \\ (x+3)^2+(y-2)^2 &= 4^2 \\ x^2+6x+9+y^2-4y+4 &= 16 \\ x^2+y^2+6x-4y-3 &= 0 \end{aligned}$$



**(b)** A circle has equation $x^2+y^2-2x+4y-15=0$. Find the values of $m$ for which the line $mx+2y-7=0$ is a tangent line.

Re-write this equation as $x^2+y^2+2(-1)x+2(2)y-15=0$ which matches the equation of a circle with $g=-1, f=2, c=-15$. So this circle has centre $(-g,-f)=(1,-2)$ and radius $r=\sqrt{g^2+f^2-c}=\sqrt{1+4+15}=\sqrt{20}$.

For a line to be tangent to this circle, the perpendicular distance from that line to the centre $(1,-2)$ must be equal to the radius. The distance from the line $mx+2y-7=0$ to $(1,-2)$ is

$$\frac{|m(1)+2(-2)-7|}{\sqrt{m^2+2^2}}$$

This must be equal to the radius in order to be tangent which means

$$\begin{aligned} \frac{|m(1)+2(-2)-7|}{\sqrt{m^2+2^2}} &= \sqrt{20} \\ |m-4-7| &= \sqrt{20}\sqrt{m^2+4} \\ |m-11| &= \sqrt{20m^2+80} \\ (m-11)^2 &= 20m^2+80 \\ m^2-22m+121 &= 20m^2+80 \\ 0 &= 19m^2+22m-41 \end{aligned}$$

We can solve this quadratic to get solutions $m=1$ and $m=-\dfrac{41}{19}$

Question 4

| Circle | Centre | Radius | Equation |
| --- | --- | --- | --- |
| $c_1$ | $(-3, -2)$ | 2 | $(x+3)^2+(y+2)^2=4$ **OR** $x^2+y^2+6x+4y+9=0$ |
| $c_2$ | $(1, 1)$ | 3 | $x^2+y^2-2x-2y-7=0$ |

**(b)** **(i)** Find the co-ordinates of the point of contact of $c_1$ and $c_2$.

Divide line segment joining $(-3, -2)$ and $(1, 1)$ in ratio 2 : 3

$$\left(\frac{2(1)+3(-3)}{2+3}, \frac{2(1)+3(-2)}{2+3}\right)=\left(-\frac{7}{5}, -\frac{4}{5}\right)$$

**OR**

Slope line of centres $=\frac{3}{4}$.

Equation line of centres: $y-1=\frac{3}{4}(x-1)\Rightarrow 3x-4y+1=0$

$c_1-c_2=4x+3y+8=0$

$4x+3y+8=0\cap 3x-4y+1=0\Rightarrow x=-\frac{7}{5}, y=-\frac{4}{5}$

**(ii)** Hence, or otherwise, find the equation of the tangent, $t$, common to $c_1$ and $c_2$.

Slope of line of centres: $\dfrac{1+2}{1+3} = \dfrac{3}{4}$

Slope of tangent: $m = -\dfrac{4}{3}$

Equation of tangent: $y + \frac{4}{5} = -\frac{4}{3}\left(x + \frac{7}{5}\right)$

$\Rightarrow \quad 3y + \frac{12}{5} = -4x - \frac{28}{5}$

$\Rightarrow \quad 4x + 3y + 8 = 0$

**OR**

$c_1 - c_2 = x^2 + y^2 + 6x + 4y + 9 - (x^2 + y^2 - 2x - 2y - 7) = 0$

$\Rightarrow 6x + 4y + 9 - (-2x - 2y - 7) = 0$

$\Rightarrow 8x + 6y + 16 = 0 \Rightarrow 4x + 3y + 8 = 0$

**OR**

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$x(-\dfrac{7}{5}) + y(-\dfrac{4}{5}) + 3(x + (-\dfrac{7}{5})) + 2(y + (-\dfrac{4}{5}) + 9 = 0$

$\Rightarrow 4x + 3y + 8 = 0$