

2nd Year Booklet 2018

Part 1



Topics covered:

- 1. Algebra**
- 2. Factors**
- 3. Algebraic Fractions**
- 4. Applied Arithmetic**

Topic 1: Algebra

In this section you must be able to:

1. Simplifying expressions
2. Removing brackets
3. Evaluating expressions
4. Solving linear equations
5. Form linear equations from word problems and solve

Key words

expression term variable coefficient constant like terms
brackets index indices linear equations number line
natural number integer rational numbers real numbers
inequalities ($>$ $>>$ $<$ $<<$)

Section 1.1 Simplifying expressions

$2x^2 + 3x - 4$ is called an **expression**.

It consists of 3 **terms**.

The terms are separated by a plus (+) or a minus (−) sign.

The letter x is called a **variable**.

A **coefficient** is a number before a variable.

In $2x^2$, the coefficient is 2; in $3x$, the coefficient is 3.

The term -4 is known as a **constant**; it does not change.

In the expression $2x^2 + 5x - 2 + 3x + 4x^2$,

$2x^2$ and $4x^2$ are called **like terms**;

$5x$ and $3x$ are also like terms.

The expression $2x^2 + 5x - 2 + 3x + 4x^2$ may be simplified by combining the like terms as follows:

$$\begin{aligned} & 2x^2 + 5x - 2 + 3x + 4x^2 \\ = & 2x^2 + 4x^2 + 5x + 3x - 2 \\ = & 6x^2 + 8x - 2 \end{aligned}$$

Like terms only may be added or subtracted.

Example 1

Simplify (i) $4a + 6b + 6 - 2a + b - 3$

$$\begin{aligned} \text{(i)} \quad & 4a + 6b + 6 - 2a + b - 3 \\ & = 4a - 2a + 6b + b + 6 - 3 \\ & = 2a + 7b + 3 \end{aligned}$$

(ii) $2x^2 - 3x - 7 - x^2 - 5x + 3$

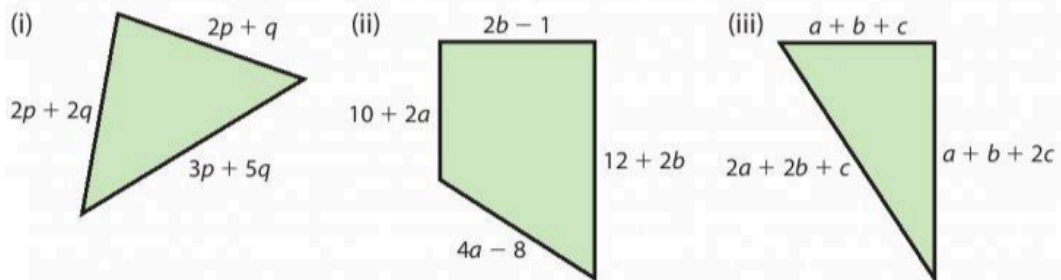
$$\begin{aligned} \text{(ii)} \quad & 2x^2 - 3x - 7 - x^2 - 5x + 3 \\ & = 2x^2 - x^2 - 3x - 5x - 7 + 3 \\ & = x^2 - 8x - 4 \end{aligned}$$

Exercise 1.1

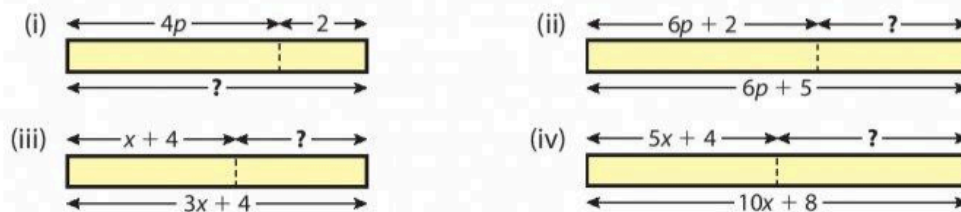
Simplify each of the following by adding like terms:

1. $3x + 4x - 2x$
 2. $7a + 3 + 4a + 6$
 3. $5x + y - 2x + 4y$
 4. $5a + 2b - 2a - 4b$
 5. $12a + b + 3a + 5b$
 6. $3x + 2y + 3 + 4x + 3y + 1$
 7. $5x - 4 + 2x + 8$
 8. $7x - 4 - 3x + 7$
 9. $6a + b + 3 + 2a + 2b - 1$
 10. $3x + 4 + 2x - 6 + x + 3$
 11. $3a - b + 4a + 5b - 2a$
 12. $2ab + 4 + 3ab - 2$
 13. $2p + 3q - r + p - 4q + 2r$
 14. $5k + 3 - 4k + 6 + k - 4$
 15. $2ab + c + 5ab - 4c$
 16. $3xy + 2z + xy + 9z$
 17. $6ab + 2cd - ab + 3cd$
 18. $6x - xy + 5x - 7xy$
 19. $x^2 - 3x + 4 - 2x^2 + 5x - 3$
 20. $3x^2 - 3x + x^2 - 8x + 7$
 21. $3a^2 - 2a - 6a + 4a^2 - 3$
 22. $y^2 - 8y - 3y^2 + 2y - 3$
 23. $3x^2 - 2 + 5x - 4 - 7x + 1$
 24. $5a^2 + 2a - 3a^2 + 4 - 3a + 2$
25. $3x^2 - 2x + 4xy + 8$ is an expression.
- (i) How many terms are there in the expression?
 - (ii) What is the coefficient of xy in the expression?
 - (iii) How many variables are there in the expression?
 - (iv) What is the constant in the expression?

26. Work out and simplify an expression for the perimeter of each of these.



27. Work out an expression for each length marked ?.



Section 1.2 Removing brackets

To write $3x \times 4y$ as a single term, we multiply the coefficients 3 and 4 and then multiply the variables x and y .

Thus, $3x \times 4y = 3 \times 4 \times x \times y = 12xy$.

Similarly, (i) $3a \times 6b = 18ab$ (ii) $-3p \times 4q = -12pq$

In your study of algebra so far, you will have learned how to remove brackets as follows:

$$a(b + c) = ab + ac \quad \text{Here each term inside the bracket is multiplied by } a.$$

Similarly, (i) $5(2x + 6) = 10x + 30$
(ii) $-4(5a - 2) = -20a + 8$
(iii) $-(6x - 4) = -6x + 4$
(iv) $-(-9a + 6) = 9a - 6$

If there is a minus outside the brackets, the sign of each term inside the brackets is changed when the brackets are removed.

Multiplication involving indices

As 5×5 can be written as 5^2 , similarly $x \times x$ is written as x^2 .

Also, $a \times a \times a = a^3$

and $a^2 \times a^2 = a \times a \times a \times a = a^4$

$$a^2 \times a^3 = a \times a \times a \times a \times a = a^5$$

$$3x(2x^2 - x + 4) = 6x^3 - 3x^2 + 12x$$

When multiplying powers of the same number, add the indices.

Multiplying two expressions

Here is a reminder of how to multiply two expressions, each containing two terms:

$$\begin{aligned}(2x + 3)(3x - 5) &= 2x(3x - 5) + 3(3x - 5) \\ &= 6x^2 - 10x + 9x - 15 \\ &= 6x^2 - x - 15\end{aligned}$$

Example 1

Remove the brackets and simplify each of these:

$$(i) (2x - 3)(3x - 4) \quad (ii) (2x - 4)(x^2 - 3x + 5)$$

$$\begin{aligned}(i) (2x - 3)(3x - 4) &= 2x(3x - 4) - 3(3x - 4) \\ &= 6x^2 - 8x - 9x + 12 \\ &= 6x^2 - 17x + 12\end{aligned}$$

$$\begin{aligned}(ii) (2x - 4)(x^2 - 3x + 5) &= 2x(x^2 - 3x + 5) - 4(x^2 - 3x + 5) \\ &= 2x^3 - 6x^2 + 10x - 4x^2 + 12x - 20 \\ &= 2x^3 - 6x^2 - 4x^2 + 10x + 12x - 20 \\ &= 2x^3 - 10x^2 + 22x - 20\end{aligned}$$

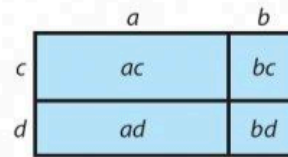
Here is a diagram to show that $(a + b)(c + d) = ac + ad + bc + bd$.

The area of the given rectangle is

$$(a + b)(c + d)$$

The rectangle is divided into 4 smaller rectangles and the area of each rectangle is shown.

This diagram illustrates that $(a + b)(c + d) = ac + ad + bc + bd$.



Exercise 1.2

1. Express each of the following as a single term:

(i) $3 \times 4a$

(ii) $3a \times 5a$

(iii) $2a \times b$

(iv) $3x \times 4y$

(v) $-2a \times 3a$

(vi) $2ab \times 4b$

(vii) $2a^2b \times ab$

(viii) $4 \times a \times 6 \times a^2$

Remove the brackets and simplify each of these:

2. $3(2x - 1) + 5(x + 2)$

3. $2(x - 4) + 3(2x + 5)$

4. $5(3x - 2) - 2(x - 1)$

5. $3(3x + 2) - 4(2x + 1)$

6. $6(2x - 3) + 2(3x - 1)$

7. $5(x - 2) - (2x + 4)$

8. $3(2a - 7) - 5(a - 4)$

9. $2(3a - 4) - (5a - 3)$

10. $2(x^2 - 3x + 1) + 2(x^2 + x - 4)$

11. $5(x^2 - x - 4) - 2(2x^2 - 3x + 2)$

Find the product of each of these:

12. $(2x + 4)(x + 3)$

13. $(3x + 2)(2x + 4)$

14. $(3x + 1)(2x - 4)$

15. $(5x - 2)(2x + 3)$

16. $(2x - 3)(x - 5)$

17. $(5x + 1)(3x - 3)$

18. $(3x + 1)(x^2 + 2x + 1)$

19. $(2x + 4)(2x^2 - x - 3)$

20. $(5x - 2)(x^2 - x + 4)$

21. $(3x - 2)(2x^2 - 3x + 7)$

22. $(4x - 1)(x^2 - 4x + 6)$

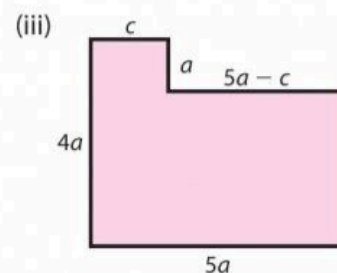
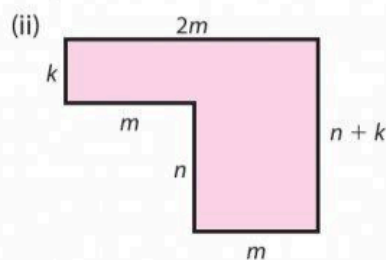
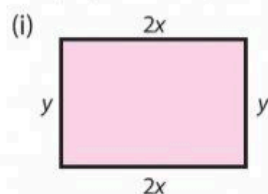
23. $(4x - 3)(2x^2 - 3x + 2)$

24. $(3a - 4)(2a^2 - 3a + 2)$

25. $(2y - 7)(y^2 + 3y + 1)$

26. Find an expression for the area of each of these shapes.

Simplify each expression fully.



27. Find the missing expressions in these statements.

(i) $d(\square) = d^2 + 5d$

(ii) $2n(\square) = 2n^2 - 8n$

(iii) $3p(\square) = 15p - 3p^2$

(iv) $5k(\square) = 20k + 5k^2$

28. Find the missing term in each statement.

(i) $5p \times \square = 10pq$

(ii) $\square \times 7n = 21mn$

(iii) $3a^2 \times \square = 9a^2b$

(iv) $\square \times 2y^2 = 10x^2y^2$

(v) $3cd^2 \times \square = 15c^3d^2$

(vi) $\square \times 7vw^3 = 28v^3w^4$

Section 1.3 Evaluating expressions

When $x = 3$, the value of $2x + 5$ is $2(3) + 5 = 6 + 5 = 11$

When $x = 2$, the value of $3x^2 - 6x$ is $3(2)^2 - 6(2) = 3(4) - 12 = 12 - 12 = 0$

Example 1

If $a = 2$, $b = 3$ and $c = -4$, find the value of

(i) $a + b$

(ii) $2a - b$

(iii) $3a^2 - 2c$

(iv) $2c^2 - 4ab$

(i) $a + b = 2 + 3 = 5$

(ii) $2a - b = 2(2) - 3$

$= 4 - 3 = 1$

(iii) $3a^2 - 2c = 3(2)^2 - 2(-4)$

$= 3(4) + 8$

$= 12 + 8 = 20$

(iv) $2c^2 - 4ab = 2(-4)^2 - 4(2)(3)$

$= 2(16) - 4(6)$

$= 32 - 24 = 8$

Remember

When evaluating an expression,

(i) multiply and divide before you add or subtract

(ii) any number multiplied by zero is zero, e.g. $6 \times 0 = 0$

Example 2

If $x = 3$ and $y = -2$, find the value of $3x^2 - 4xy - 2y^2$.

$$3x^2 - 4xy - 2y^2 = 3(3)^2 - 4(3)(-2) - 2(-2)^2$$

$$= 3(9) - 4(-6) - 2(4) \quad \dots \text{square before you multiply}$$

$$= 27 + 24 - 8$$

$$= 51 - 8$$

$$= 43$$

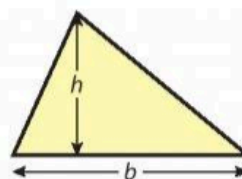
Exercise 1.3

- If $x = 4$, find the value of each of these:
 - $3x$
 - $2x + 6$
 - $7 - x$
 - $x^2 + 3$
 - $3x^2 - 2x - 7$
- If $a = 3$ and $b = -2$, find the value of each of these:
 - $2a + b$
 - $a^2 + ab$
 - $2a^2 + b^2$
 - $3ab + 2b^2$
- If $a = 1, b = 2$ and $c = 3$, find the value of:
 - $2a + b$
 - $3ab - c$
 - $4abc + 3c$
 - $3bc - 4ab$
 - $3abc - 2ac$
 - $5bc - 2ab$
 - $\frac{3a + 6b}{c}$
 - $\frac{4b - 2a}{c}$
 - $\frac{6c - ab}{4b}$
- If $x = 1\frac{1}{2}$ and $y = \frac{1}{2}$, find the value of:
 - $4x + 2y$
 - $3x + y$
 - $2x - 4y$
 - $8xy$
- If $a = 4$ and $b = 3$, find the value of:
 - $a^2 + b$
 - $b^2 - 2a$
 - $3a^2 - 2b^2$
 - $a^2b - 2ab$
- If $x = 2$, find the value of $2x^2 - 3x + 5$.
- If $a = 5, b = 2$ and $c = 8$, evaluate each of these:
 - $a - bc$
 - $2a^2 - bc$
 - $a(b - c)^2$
- Calculate the value of each of the following expressions when $p = 2, q = 3$ and $r = 6$:
 - $r(p + q)$
 - $rp + q$
 - $\frac{r}{pq}$
 - $\frac{q + r}{p}$
 - $5r^2$
- If $x = 2, y = -3$ and $z = 1$, evaluate
 - $x^2 - 3xy + 2z$
 - $2y^2 - 3xy$
 - $z^2 - 3x^2y + 4y$

- 10.** The area of a triangle is given by the formula $A = \frac{bh}{2}$.

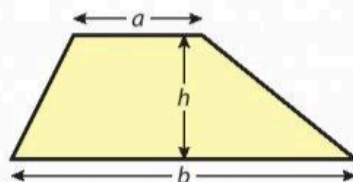
Work out the areas of triangles where

- $b = 25$ cm, $h = 10$ cm
- $b = 16$ cm, $h = 12$ cm



- 11.** The area, A square units, of a trapezium is given by the formula $A = \frac{1}{2}(a + b)h$. (a, b and h are shown on the diagram.) Use the formula to calculate the areas of trapeziums for which

- $a = 9$ cm, $b = 16$ cm, $h = 10$ cm
- $a = 14$ cm, $b = 30$ cm, $h = 11$ cm



12. When $m = -1$, $n = -2$ and $p = 8$, there are four matching pairs here. Can you find them?

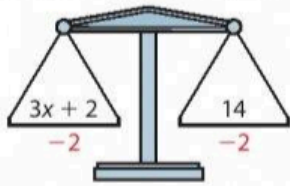
A mp
B $mp - n$
C $2mn - p$
D $n + p$
E $\frac{p}{n}$
F $4n$
G $\frac{p}{m} - n$
H $-3n$

13. Evaluate $\frac{3}{x+3} + \frac{4}{2x+6}$, when $x = \frac{1}{3}$.

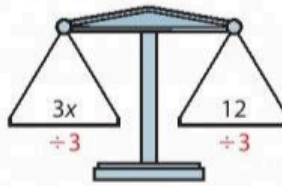
Section 1.4 Solving linear equations

The three scales below will help you recall the steps involved in solving an equation.

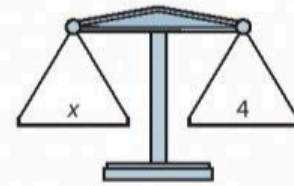
We need to solve the equation $3x + 2 = 14$.



Take 2 from both sides



Divide both sides by 3



$x = 4$ is the solution

The scales above illustrate that solving an equation requires us to change the equation into a simpler one. This is done by adding, subtracting, multiplying or dividing both sides of the equation by the **same number**.

Example 1

Solve the equation $3x - 5 = 16$

$$\begin{aligned}
 3x - 5 &= 16 \\
 3x - 5 + 5 &= 16 + 5 \quad \dots \text{add 5 to both sides} \\
 3x &= 21 \\
 x &= 7 \quad \dots \text{divide both sides by 3}
 \end{aligned}$$

Example 2

Solve the equation $3(2x - 6) = 2(2x + 1)$

$$\begin{aligned}
 3(2x - 6) &= 2(2x + 1) \\
 6x - 18 &= 4x + 2 \quad \dots \text{remove the brackets} \\
 6x - 4x - 18 &= 4x + 2 - 4x \quad \dots \text{take 4x from each side} \\
 2x - 18 &= 2 \\
 2x - 18 + 18 &= 2 + 18 \quad \dots \text{add 18 to each side} \\
 2x &= 20 \\
 x &= 10 \quad \dots \text{divide each side by 2}
 \end{aligned}$$

Exercise 1.4

1. Write down the missing numbers to make these equations true:

(i) $\square + 8 = 15$

(ii) $12 - \square = 8$

(iii) $\square \times 3 = 30$

(iv) $\square \times 6 + 2 = 26$

(v) $27 \div \square + 1 = 10$

(vi) $7 + 5 + \frac{\square}{2} = 15$

2. Solve each of these equations:

(i) $2x = 8$

(ii) $3x = 27$

(iii) $5x = 35$

(iv) $6x = 42$

(v) $9x = 63$

Solve each of these equations:

3. $4x + 2 = 14$

4. $2x - 9 = 3$

5. $2x - 7 = 5$

6. $3 + 2x = 11$

7. $6 + 3x = 9$

8. $2 + 7x = 30$

9. $6x - 2 = 4x + 10$

10. $7x - 9 = 3x + 11$

11. $9x - 15 = 3x + 3$

12. $x + 7 = 2x - 1$

13. $2x + 6 = 4x - 6$

14. $3x + 1 = 5x - 13$

15. $5x - 2 = 40 - x$

16. $3x + 7 = 32 - 2x$

17. $6 + 2x = 33 - x$

Remove the brackets and solve the following equations:

18. $3(2x + 1) = 2x + 11$

19. $2(2x + 5) = 5x + 5$

20. $4(2x - 3) = 2(3x - 5)$

21. $4(3x + 6) = 3(5x - 2)$

22. $5(2x - 4) + 1 = 3(2x - 1)$

23. $6(2x + 1) + 4 = 5(3x - 1)$

24. $5(x - 2) - 3x = 3(x - 5)$

25. $5(2x + 3) = 4(2x + 1) + 15$

26. $5(x + 3) - 25 = 6(2 - x)$

27. $10(x + 4) - 1 = 3(2x + 5)$

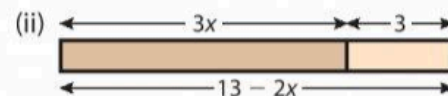
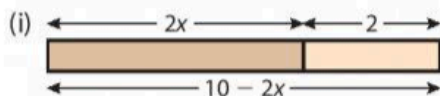
28. $4(x - 2) - 9 = 3 - (x + 5)$

29. $3(x - 1) = 18 - 5(x + 1)$

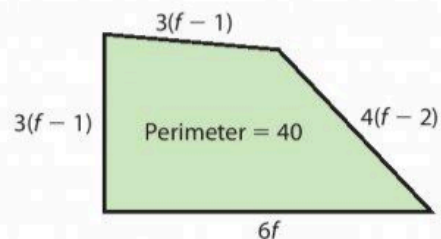
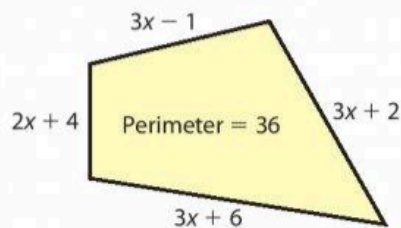
30. $3(5x + 7) + 2(3x - 5) = 5(2x + 11)$

31. $5(3x - 2) = 3(2x + 1) + 2x + 1$

32. Work out what x stands for in each of these.



33. Find the values of x and f in these figures. The perimeter is given in each case.



OL Wordy Algebra

Section 2.5 Writing and solving equations

The ability to change a problem expressed in words into a mathematical equation and then to solve it is very important in mathematics.

The following example will show how equations can be formed from word problems.

Example 1

When 3 is taken from five times a certain number, the result is the same as adding 6 to twice the number. Find the number.

Let x be the required number.

Equation: $5x - 3 = 2x + 6$

$$5x - 3 + 3 = 2x + 6 + 3 \quad \dots \text{add 3 to each side}$$

$$5x = 2x + 9$$

$$5x - 2x = 2x + 9 - 2x \quad \dots \text{take } 2x \text{ from each side}$$

$$3x = 9$$

$$x = 3$$

The required number is 3.

Exercise 2.5

1. Write an equation for each of the following and solve it.

Let x be the unknown number in each case.

(i) I think of a number.
When I multiply it by 3 and then add 2, the answer is 17.
What is the number?

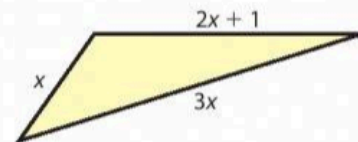
(ii) I think of a number.
When I multiply it by 4 and then add 1, the answer is 13.
What is the number?

(iii) I think of a number.
When I multiply the number by 3 and then subtract 4, the answer is the same as twice the number.
What is the number?

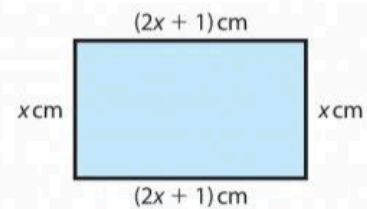
(iv) I think of a number.
When I add 2 to the number and multiply my answer by 4, the result is 20.
What is the number?

Text & Tests 2 Ordinary Level

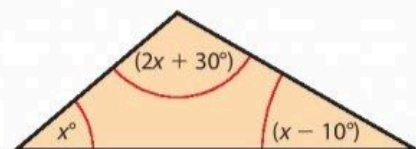
2. If I multiply a number by 4 and then add 3, the result is the same as adding 8 to three times the number.
Form an equation in x and solve it to find the number.
3. I think of a number, multiply it by 8, and then subtract 2.
I get the same result as when I multiply this number by 2 and add 10.
What is this number?
4. A number is multiplied by 3 and 5 is then added.
If the result is 17, find the number.
5. When 3 is added to a number and the result is doubled, the result is 24.
Write an equation in x and solve it to find the number.
6. 9 is subtracted from a certain number and the result is multiplied by 4.
If the answer is 24, what is the number?
7. One number is 4 bigger than another number.
If x is the smaller number, what is the larger one?
When twice the smaller number is added to the larger, the result is 16.
Find the two numbers.
8. One number is 5 greater than another number.
If the smaller number is added to twice the larger number, the answer is 28.
Find the two numbers.
9. (i) Find an expression for the perimeter of this triangle.
(ii) What value of x gives a perimeter of 55?



10. If we subtract 4 from a number and then multiply the result by 5, the answer is 15.
Find the number.
11. (i) Find an expression, in terms of x , for the perimeter of this rectangle.
Give your answer in its simplest form.
(ii) The perimeter of the rectangle is 44 cm.
Write down an equation and solve it to find the value of x .

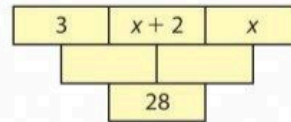


12. (i) Write an expression for the sum of the angles marked in this triangle.
Give your answer in its simplest form.
(ii) The angles of a triangle add up to 180° .
Write down an equation in x and use it to find the value of x .

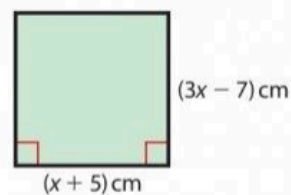


- 13.** In a furniture store, a chair costs € x and a stool costs €10 less than a chair. If 2 stools and 3 chairs cost €230, find the cost of a chair and the cost of a stool.

- 14.** In this diagram, the number in each box is found by adding the two numbers above it. Find the value of x .



- 15.** The given figure is a square. Form an equation in x and solve it. Hence, find the length of the side of the square.

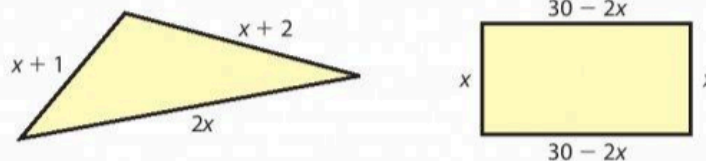


- 16.** Emer is 4 years older than Leah. If twice the sum of their ages is 48 years, how old is Leah?

- 17.** 166 people live in an apartment block and of these, x are women. There are 8 fewer men than women, and there are 30 more children than women. How many women live in the block?



- 18.** A triangle and rectangle are shown below:



Find the value of x which

- gives a triangle with a perimeter of 63
- gives a triangle and rectangle with equal perimeters
- makes the rectangle into a square.

Section 1.5 Solving problems using linear equations —

The ability to change a problem expressed in words into a mathematical equation and then solve it is very important in mathematics.

The following examples will show how equations can be formed from word problems.

Example 1

When five times a certain number is reduced by 4, the result is the same as adding 7 to four times the number.

Find the number.

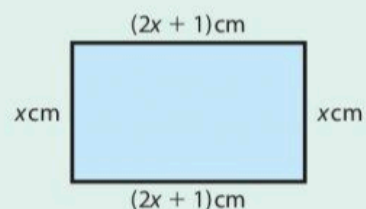
Let x be the required number.

$$\begin{aligned} \text{Equation:} \quad & 5x - 4 = 4x + 7 \\ & 5x - 4 - 4x = 4x + 7 - 4x \quad \dots \text{ take } 4x \text{ from each side} \\ & \quad x - 4 = 7 \\ & x - 4 + 4 = 7 + 4 \quad \dots \text{ add } 4 \text{ to each side} \\ & \quad x = 11 \end{aligned}$$

The required number is 11.

Example 2

- (i) Find an expression, in terms of x , for the perimeter of this rectangle.
Give your answer in its simplest form.
- (ii) The perimeter of the rectangle is 44 cm.
Write down an equation and solve it to find the value of x .

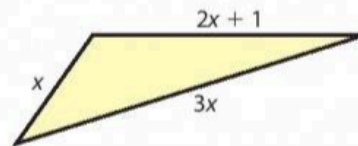


- (i) Perimeter = $(2x + 1) + x + (2x + 1) + x$
 $= (6x + 2) \text{ cm}$
- (ii) Perimeter = 44 $\Rightarrow x + 2 = 44$
 $6x + 2 - 2 = 44 - 2$
 $6x = 42$
 $x = 7$

Exercise 1.5

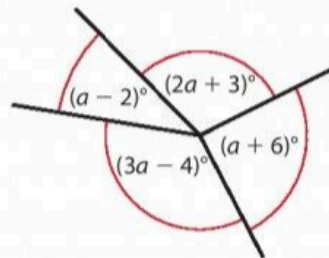
- If I multiply a number by 4 and then add 3, the result is the same as adding 8 to three times the number.
Form an equation in x and solve it to find the number.
- I think of a number, multiply it by 8 and then subtract 2.
I get the same result as when I multiply this number by 2 and add 10. What is this number?
- One number is 5 greater than another number.
If the smaller number is added to twice the larger number, the answer is 28.
Find the two numbers.
- Ann is 3 years older than Helen.
If twice the sum of their ages is 50 years, how old is Ann?

- (i) Find an expression for the perimeter of this triangle.
(ii) What value of x gives a perimeter of 55?

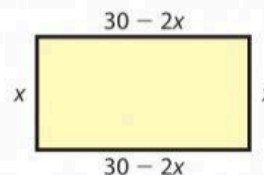
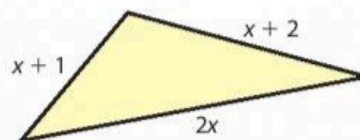


- If we subtract 4 from a number and then multiply the result by 5, the answer is 15.
Find the number.
- I think of a number, increase it by 4 and double the answer.
The result is 20 more than the number. Find this number.

- Use your knowledge of angles to form an equation and solve it to find the value of a .



- A triangle and rectangle are shown below:



Find the value of x which

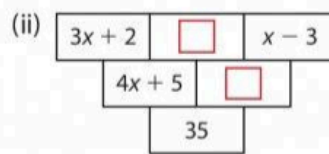
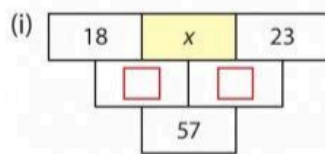
- gives a triangle with a perimeter of 63
- gives a triangle and rectangle with equal perimeters
- makes the rectangle into a square.

10. Here is a fraction: $\frac{x}{\square}$.

The denominator of the fraction is 5 more than the numerator.

If 1 is added to the numerator and 2 is subtracted from the denominator, the fraction will become $\frac{4}{5}$. Find the fraction.

11. The number in each brick is found by adding the two numbers above it. Find the missing expressions in each of the diagrams below. Write equations to find the value of x in each case.

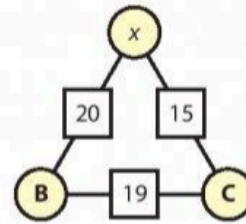


12. In an arithmagon, the number in a square is the sum of the numbers in the two circles either side of it.

(i) Explain why the number in circle **B** is $20 - x$.

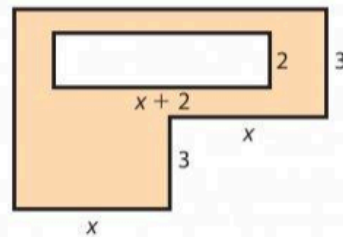
(ii) What is the number in circle **C** in terms of x ?

(iii) Form an equation across the base of the triangle and solve it to find the value of x .



13. Find an expression in x for the area of the shaded portion of this figure.

If the area of this shaded portion is 38 cm^2 , find the value of x .



14. Ruth is x years old. Write, in terms of x ,

(i) her age 6 years ago

(ii) her age in 12 years time.

In 12 years time, Ruth will be three times as old as she was 6 years ago.

Find Ruth's age now.

15. 166 people live in an apartment block and of these, x are women.

There are 8 fewer men than women and there are 30 more children than women.

How many women live in the block?

Answers

Chapter 1: Algebra 1

Exercise 1.1

- | | |
|----------------------|----------------------|
| 1. $5x$ | 2. $11a + 9$ |
| 3. $3x + 5y$ | 4. $3a - 2b$ |
| 5. $15a + 6b$ | 6. $7x + 5y + 4$ |
| 7. $7x + 4$ | 8. $4x + 3$ |
| 9. $8a + 3b + 2$ | 10. $6x + 1$ |
| 11. $5a + 4b$ | 12. $5ab + 2$ |
| 13. $3p - q + r$ | 14. $2k + 5$ |
| 15. $7ab - 3c$ | 16. $4xy + 11z$ |
| 17. $5ab + 5cd$ | 18. $11x - 8xy$ |
| 19. $-x^2 + 2x + 1$ | 20. $4x^2 - 11x + 7$ |
| 21. $7a^2 - 8a - 3$ | 22. $-2y^2 - 6y - 3$ |
| 23. $3x^2 - 2x - 5$ | 24. $2a^2 - a + 6$ |
| 25. (i) 4 | (ii) 4 |
| (iii) 2 | (iv) 8 |
| 26. (i) $7p + 8q$ | (ii) $6a + 4b + 13$ |
| (iii) $4a + 4b + 4c$ | |
| 27. (i) $4p + 2$ | (ii) 3 |
| (iii) $2x$ | (iv) $5x + 4$ |

Exercise 1.2

- | | | |
|-------------------------------|-----------------------|--------------|
| 1. (i) $12a$ | (ii) $15a^2$ | (iii) $2ab$ |
| (iv) $12xy$ | (v) $-6a^2$ | (vi) $8ab^2$ |
| (vii) $2a^3b^2$ | (viii) $24a^3$ | |
| 2. $11x + 7$ | 3. $8x + 7$ | |
| 4. $13x - 8$ | 5. $x + 2$ | |
| 6. $18x - 20$ | 7. $3x - 14$ | |
| 8. $a - 1$ | 9. $a - 5$ | |
| 10. $4x^2 - 4x - 6$ | 11. $x^2 + x - 24$ | |
| 12. $2x^2 + 10x + 12$ | 13. $6x^2 + 16x + 8$ | |
| 14. $6x^2 - 10x - 4$ | 15. $10x^2 + 11x - 6$ | |
| 16. $2x^2 - 13x + 15$ | 17. $15x^2 - 12x - 3$ | |
| 18. $3x^3 + 7x^2 + 5x + 1$ | | |
| 19. $4x^3 + 6x^2 - 10x - 12$ | | |
| 20. $5x^3 - 7x^2 + 22x - 8$ | | |
| 21. $6x^3 - 13x^2 + 27x - 14$ | | |
| 22. $4x^3 - 17x^2 + 28x - 6$ | | |
| 23. $8x^3 - 18x^2 + 17x - 6$ | | |
| 24. $6a^3 - 17a^2 + 18a - 8$ | | |
| 25. $2y^3 - y^2 - 19y - 7$ | | |
| 26. (i) $2xy$ | (ii) $2km + nm$ | |
| (iii) $15a^2 + ac$ | | |
| 27. (i) $\boxed{d + 5}$ | (ii) $\boxed{n - 4}$ | |
| (iii) $\boxed{5 - p}$ | (iv) $\boxed{4 + k}$ | |

- | | | |
|--------------|------------|--------------|
| 28. (i) $2q$ | (ii) $3m$ | (iii) $3b$ |
| (iv) $5x^2$ | (v) $5c^2$ | (vi) $4v^2w$ |

Exercise 1.3

- | | | | | |
|--|-------------------------|-----------|--------------------|---------|
| 1. (i) 12 | (ii) 14 | (iii) 3 | (iv) 19 | (v) 33 |
| 2. (i) 4 | (ii) 3 | (iii) 22 | (iv) -10 | |
| 3. (i) 4 | (ii) 3 | (iii) 33 | (iv) 10 | (v) 12 |
| (vi) 26 | (vii) 5 | (viii) 2 | (ix) 2 | |
| 4. (i) 7 | (ii) 5 | (iii) 1 | (iv) 6 | |
| 5. (i) 19 | (ii) 1 | (iii) 30 | (iv) 24 | |
| 6. 7 | | | | |
| 7. (i) -11 | (ii) 34 | (iii) 180 | | |
| 8. (i) 30 | (ii) 15 | (iii) 1 | (iv) $\frac{9}{2}$ | (v) 180 |
| 9. (i) 24 | (ii) 36 | (iii) 25 | | |
| 10. (i) 125 cm^2 | (ii) 96 cm^2 | | | |
| 11. (i) 125 cm^2 | (ii) 242 cm^2 | | | |
| 12. A and F, B and G, C and E, D and H | | | | |
| 13. $1\frac{1}{2}$ | | | | |

Exercise 1.4

- | | | |
|---|--------------|---------------|
| 1. (i) 7 | (ii) 4 | (iii) 10 |
| (iv) 4 | (v) 3 | (vi) 6 |
| 2. (i) $x = 4$ | (ii) $x = 9$ | (iii) $x = 7$ |
| (iv) $x = 7$ | (v) $x = 7$ | |
| 3. $x = 3$ | 4. $x = 6$ | 5. $x = 6$ |
| 6. $x = 4$ | 7. $x = 1$ | 8. $x = 4$ |
| 9. $x = 6$ | 10. $x = 5$ | 11. $x = 3$ |
| 12. $x = 8$ | 13. $x = 6$ | 14. $x = 7$ |
| 15. $x = 7$ | 16. $x = 5$ | 17. $x = 9$ |
| 18. $x = 2$ | 19. $x = 5$ | 20. $x = 1$ |
| 21. $x = 10$ | 22. $x = 4$ | 23. $x = 5$ |
| 24. $x = 5$ | 25. $x = 2$ | 26. $x = 2$ |
| 27. $x = -6$ | 28. $x = 3$ | 29. $x = 2$ |
| 30. $x = 4$ | 31. $x = 2$ | |
| 32. (i) $x = 2$ | (ii) $x = 2$ | |
| 33. $x = 2\frac{3}{11}, f = 3\frac{3}{8}$ | | |

Exercise 1.5

- | | |
|--------------------|---------------------|
| 1. 5 | 2. 2 |
| 3. 6 and 11 | 4. 14 yrs old |
| 5. (i) $6x + 1$ | (ii) 9 |
| 6. 7 | 7. 12 |
| 9. (i) 15 | (ii) $9\frac{1}{2}$ |
| (iii) 10 | |
| 10. $\frac{7}{12}$ | |

11. (i) 8 (ii) 5
 12. (i) As $20 - x + x = 20$
 (ii) $15 - x$ (iii) 8
 13. $(7x - 4) \text{ cm}^2$; 6
 14. (i) $(x - 6) \text{ yrs}$ (ii) $(x + 12) \text{ yrs}$
 (iii) 15 yrs
 15. 48
 16. Air – 2 hours, Sea – 5 hours 20 min,
 Train – 3 hours 20 min

Answers to the small OL Section 2.5 Wordy Algebra

Exercise 2.5

1. (i) $3x + 2 = 17$; 5 (ii) $4x + 1 = 13$; 3
 (iii) $3x - 4 = 2x$; 4 (iv) $4(x + 2) = 20$; 3
 2. $4x + 3 = 3x + 8$; 5
 3. 2 4. 4 5. 9
 6. 15 7. $(x + 4)$; 4, 8 8. 6, 11
 9. (i) $6x + 1$ (ii) 9
 10. 7
 11. (i) $(6x + 2) \text{ cm}$ (ii) $6x + 2 = 44$; $x = 7$
 12. (i) $(4x + 20)^\circ$
 (ii) $4x + 20 = 180^\circ$; $x = 40^\circ$
 13. Chair – €50, stool – €40
 14. $x = 7$
 15. $x + 5 = 3x - 7$; $x = 6$; 11 cm
 16. 10 years old
 17. 48
 18. (i) 15 (ii) 9.5 (iii) 10

Topic 2: Factors

In this section you must be able to:

- 1. Get the Highest common factor**
- 2. Group Factors**
- 3. Difference of 2 squares**
- 4. Factorise Quadratic equations**
- 5. Simplifying fractions using factors**

Key words

factor highest common factor grouping terms difference of two squares
quadratic expressions

Section 4.1 Factorising with common factors

Since $9 \times 5 = 45$, we say that 9 and 5 are **factors** of 45.

15 and 3 are also factors of 45.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.

The **highest common factor** is 12.

Here are two algebraic terms: $6xy$ and $12x$.

The highest common factor of the numbers is 6.

The highest common factor of the variables is x .

So the highest common factor of the two terms is $6 \times x$, i.e. $6x$.

Similarly, the highest common factors of

(i) $3a$ and $6a^2 = 3a$

(ii) $6x^2 - 12xy = 6x$

(iii) $5a^2b - 15ab = 5ab$

(iv) $4x^2 + 16xy^2 = 4x$

Take the expression $5x + 10$.

$$5x + 10 = 5(x + 2)$$

5 and $(x + 2)$ are called the factors of $5x + 10$.

To factorise an algebraic expression:

- Find the highest common factor and write it outside the brackets.
- Divide each term by this factor and write the results inside the brackets.
- Check your result by expanding the brackets.

Here are some expressions that have been factorised:

(i) $x^2 + 7x = x(x + 7)$

(ii) $3x^2 - 9x = 3x(x - 3)$

(iii) $3xy - 12x = 3x(y - 4)$

(iv) $12x^2y^2 - 6xy = 6xy(2xy - 1)$

Exercise 4.1

1. Write down the highest common factor of each of these:

- (i) 9 and 12 (ii) 12 and 18 (iii) 14 and 21 (iv) 21 and 35

2. Write down the highest common factor of each of these:

- (i) $4x$ and $12x$ (ii) $3n$ and $9n$ (iii) $10x$ and $15x$
 (iv) $3a^2$ and $6a$ (v) $3xy$ and $12x^2$ (vi) $2a^2b$ and $6ab$

Copy and complete numbers (3–16).

3. $4x + 8 = 4(\quad)$ 4. $6a + 12b = 6(\quad)$
 5. $7x + 14y = 7(\quad)$ 6. $22a + 33b = 11(\quad)$
 7. $12x - 24y = 12(\quad)$ 8. $ab + bc = b(\quad)$
 9. $2ax + 4ay = 2a(\quad)$ 10. $x^2 + x = x(\quad)$
 11. $5x^2 - 10x = 5x(\quad)$ 12. $3a^2 + 6a = 3a(\quad)$
 13. $4x^2 - 12x = 4x(\quad)$ 14. $6ab - 12bc = 6b(\quad)$
 15. $7x^2y - 14xy = 7xy(\quad)$ 16. $6x^2y - 15x^2 = 3x^2(\quad)$

Factorise each of the following:

17. $4x + 16$ 18. $6x + 18y$ 19. $10a - 40b$
 20. $ab + ay$ 21. $3ab + 3bc$ 22. $2xy - 2xz$
 23. $7xy + 14xz$ 24. $5ab - 15bc$ 25. $6ax - 12ay$
 26. $x^2 + 4x$ 27. $3x^2 + 9x$ 28. $5x^2 + 10x$
 29. $6a^2 - 12a$ 30. $7a^2 - 21a$ 31. $10x^2 + 40x$
 32. $25x^2 - 15x$ 33. $7x^2 - 28x$ 34. $12x^2 + 18x$
 35. $15x^2 + 25xy$ 36. $3x^2 - 6x^2y$ 37. $x^3 + x^2 + x$
 38. $3ab^2 - 6ab$ 39. $12xy - 24x^2y$ 40. $15x^3 - 35x^2$

41.

E	H	P	S	O	A	I	L	G	R	T	U	N
5	$2a$	$3a$	$2b$	$7b$	a^2	ab	$3b^2$	$a + b$	$a - 5b$	$2a - b$	$ab + 1$	$2a + 3b$

HL FACTORS

Exercise 2.1

- Write down the highest common factor of each of these:
 - 9 and 12
 - 12 and 18
 - 14 and 21
 - 21 and 35
- Write down the highest common factor of each of these:
 - $4x$ and $12x$
 - $3n$ and $9n$
 - $10x$ and $15x$
 - $3a^2$ and $6a$
 - $3xy$ and $12x^2$
 - $2a^2b$ and $6ab$
- Copy and complete each of these:
 - $7x + 14y = 7(\quad)$
 - $16a + 24b = 8(\quad)$
 - $ab + bc = b(\quad)$
 - $3a^2 + 6a = 3a(\quad)$
 - $5x^2 - 15xy = 5x(\quad)$
 - $12xy - 18yz = 6y(\quad)$
 - $15x^3 + 10x^2y = 5x^2(\quad)$
 - $6a^2b - 8ab^2 + 4ab = 2ab(\quad)$

Factorise each of the following:

- $6x + 18y$
- $3ab + 3bc$
- $6ax - 12ay$
- $6a^2 - 12a$
- $7x^2 - 28x$
- $15x^2 + 25xy$
- $3x^2 - 6x^2y$
- $3ab^2 - 6ab$
- $3p^2 - 6pq$
- $2x^2y - 6x^2z$
- $6y^2z + 10y^2$
- $10p^2q + 5pq^2$
- $2a^3 - 4a^2 + 8a$
- $4x^2 - 6xy + 8xz$
- $5xy^2 - 20x^2y$
- $4x^2y^2 - 8xy$
- $5x^3 - 10x^2 + 15x$
- $2a^2b - 4ab^2 + 12abc$

22.

E	H	P	S	O	A	I	L	G	R	T	U	N
5	$2a$	$3a$	$2b$	$7b$	a^2	ab	$3b^2$	$a + b$	$a - 5b$	$2a - b$	$ab + 1$	$2a + 3b$

Fully factorise each expression below as the product of two factors.

Use the code above to find a letter for each factor.

Rearrange each set of letters to spell a bird.

- $3a^2 - 15ab$ $2a^3 - a^2b$ $7ab - 35b^2$
- $4a^2 - 2ab$ $2a^2b + 2a$ $2ab - 10b^2$
- $7ab + 7b^2$ $5a - 25b$ $2ab^2 + 2b$
- $4ab - 2b^2$ $3b^2a + 3b^3$ $a^3 - 5a^2b$ $2a^2b + 3ab^2$

OL FACTORISING BY GROUPING

Section 4.2 Factorising by grouping terms

Some four-termed expressions do not have an overall common factor, but can be factorised by pairing the four terms.

For example,
$$\begin{aligned} & \underbrace{ab + ac} + \underbrace{bd + dc} \\ &= a(b + c) + d(b + c) \quad \dots \text{factorising each pair separately} \\ &= (b + c)(a + d) \quad \dots \text{removing common factor } (b + c) \end{aligned}$$

Example 1

Find the factors of (i) $2ab + 2ac + 3bx + 3cx$

(ii) $3ax - bx - 3ay + by$

$$\begin{aligned} \text{(i)} \quad & \underbrace{2ab + 2ac} + \underbrace{3bx + 3cx} = 2a(b + c) + 3x(b + c) \\ &= (b + c)(2a + 3x) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & \underbrace{3ax - bx} - \underbrace{3ay + by} = x(3a - b) - y(3a - b) \\ &= (3a - b)(x - y) \end{aligned}$$

Note: 1. $-(x + y)$ is the same as $-1(x + y)$

So if you have $x(x + y) - (x + y)$, the factors are

$$(x + y)(x - 1)$$

2. Be careful when dealing with negative terms.

For example, (i) $-3ax - 6ay = -3a(x + 2y)$

(ii) $-5x^2 + 10xy = -5x(x - 2y)$

Example 2Factorise $6x^2 + 2a - 3ax - 4x$.

$$\begin{aligned}
 \text{Regroup: } & 6x^2 - 3ax + 2a - 4x \\
 & 3x(2x - a) + 2(a - 2x) \\
 & = 3x(2x - a) - 2(2x - a) \\
 & = (2x - a)(3x - 2)
 \end{aligned}$$

$$\therefore 6x^2 + 2a - 3ax - 4x = (2x - a)(3x - 2)$$

Here we initially had no common factor, so we changed $+2(a - 2x)$ to $-2(2x - a)$. $(2x - a)$ is now a common factor.

Exercise 4.2

Write down the factors of each of the following:

1. $a(x + 4) + b(x + 4)$
2. $x(2a + 3) + 4(2a + 3)$
3. $x(y + z) + y(y + z)$
4. $a(b + 2c) + 3(b + 2c)$
5. $a(x - 6) + 3(x - 6)$
6. $2a(x + y) - 3b(x + y)$
7. $3x(2x - 3) + (2x - 3)$
8. $2a(3b - c) - (3b - c)$

Find the factors of each of the following expressions:

9. $ax + ay + bx + by$
10. $ab + bc + ad + cd$
11. $ac + bc + 3a + 3b$
12. $mp + np + mq + nq$
13. $2ab + 2bc + 3ad + 3cd$
14. $7ax - 7bx + 3a - 3b$
15. $ac - bc + 2a - 2b$
16. $ac - bc + 2ad - 2bd$
17. $2ax + 3ay + 2bx + 3by$
18. $4x - 4y + ax - ay$
19. $3a - 3b + ax - bx$
20. $ax - 2a + 4x - 8$
21. $2ab - bc + 2ad - cd$
22. $2ax - ay + 2bx - by$
23. $x^2 + ax + bx + ab$
24. $a^2 - 2ab + ac - 2bc$
25. $4ay + xy - 4az - xz$
26. $3mx - am + 3nx - an$
27. $x^2 - xy + xz - yz$
28. $2x - 2y - cx + cy$
29. $3x + 3y - bx - by$
30. $5a + 5b - ac - bc$
31. $x^2 - 2x + xy - 2y$
32. $a^2 - 3a + ab - 3b$

33. $ax - a - bx + b$

35. $x(2a - 3b) + 2a - 3b$

37. $ab - bc + ac - b^2$

39. $3a - 3b + ab - 9$

34. $ay - 2by - az + 2bz$

36. $a(x + 2y) - x - 2y$

38. $ab - 2cd - bc + 2ad$

40. $3ax - 4by + 2bx - 6ay$

HL FACTORISING BY GROUPING

Exercise 2.2

Factorise fully each of the following:

1. $2a(x + y) + 3(x + y)$
2. $3x(2a - b) - 4(2a - b)$
3. $3a(2b - c) - 4(2b - c)$
4. $2x(5y - z) + b(5y - z)$
5. $2a(x - 2y) - (x - 2y)$
6. $a^2 + ab + ac + bc$
7. $x^2 - ax + 3x - 3a$
8. $ab + ac - 5b - 5c$
9. $ab + 5b + 3a + 15$
10. $3x^2 - 3xz + 4xy - 4yz$
11. $2c^2 - 4cd + c - 2d$
12. $2ax - 6ay - 3x + 9y$
13. $2ac - 4ad + bc - 2bd$
14. $3xy - 3xyz + 2z - 2z^2$
15. $8ax + 4ay - 6bx - 3by$
16. $6ax^2 + 9a - 8x^2 - 12$
17. $x(2y - z) - 2y + z$
18. $an - 5a - 5b + bn$
19. $2x^2y - 2xz - 3xy + 3z$
20. $7y^2 - 21by + 2ay - 6ab$
21. $4a^2b - 3b - 6a + 2ab^2$
22. $12a^2 - 8ab + 9ac - 6bc$
23. $10ab - 5ac - 2bd + cd$
24. $4x^2 + 3ay - 2ax - 6xy$
25. $6a^2 + 15xy - 10ay - 9ax$
26. $6xy + 12yz - 8xz - 9y^2$
27. $3abx^2 - 5axy - 3bxy + 5y^2$
28. $6a^2c - 6ab - 4bc + 9a^3$
29. $x^2 - x(2a - b) - 2ab$
30. $6x^2 - 3y(3x - 2a) - 4ax$

OL DIFFERENCE OF TWO SQUARES

Section 4.3 Difference of two squares

Numbers such as 1, 4, 9, 16, 25, ... are called **perfect squares** as they are obtained by multiplying some whole number by itself, e.g. $4 = 2^2$, $9 = 3^2$, ...

Similarly, in algebra, $4x^2 = (2x)^2$ and $9y^2 = (3y)^2$.

Expressions such as $10^2 - 4^2$, $x^2 - y^2$ and $4x^2 - 9$ are known as the **difference of two squares**.

When you multiply $(x + y)(x - y)$, you get $x^2 - y^2$.

Thus, the factors of $x^2 - y^2$ are $(x + y)(x - y)$.

$$x^2 - y^2 = (x + y)(x - y)$$

In words: $(\text{first})^2 - (\text{second})^2 = (\text{first} + \text{second})(\text{first} - \text{second})$

Example 1

Factorise (i) $x^2 - 9$ (ii) $x^2 - 4y^2$ (iii) $9a^2 - 25b^2$

(i) $x^2 - 9 = (x)^2 - (3)^2 = (x + 3)(x - 3)$

(ii) $x^2 - 4y^2 = (x)^2 - (2y)^2 = (x + 2y)(x - 2y)$

(iii) $9a^2 - 25b^2 = (3a)^2 - (5b)^2 = (3a + 5b)(3a - 5b)$

Example 2

Use the difference of two squares to evaluate $51^2 - 49^2$.

$$51^2 - 49^2 = (51 + 49)(51 - 49) \quad \dots \quad a^2 - b^2 = (a + b)(a - b)$$

$$= (100)(2)$$

$$= 200$$

Exercise 4.3

Factorise each of the following:

- | | | | |
|---------------------------|----------------------------|----------------------------|-------------------------|
| 1. $x^2 - 4$ | 2. $x^2 - 16$ | 3. $y^2 - 36$ | 4. $y^2 - 100$ |
| 5. $x^2 - y^2$ | 6. $x^2 - 16y^2$ | 7. $a^2 - 9b^2$ | 8. $4x^2 - y^2$ |
| 9. $9a^2 - 1$ | 10. $4x^2 - 9$ | 11. $9x^2 - 100$ | 12. $100x^2 - 1$ |
| 13. $4a^2 - 25b^2$ | 14. $36x^2 - 49y^2$ | 15. $1 - 36x^2$ | 16. $1 - 100a^2$ |
| 17. $49 - 81x^2$ | 18. $64 - 81y^2$ | 19. $64x^2 - 25y^2$ | 20. $x^2y^2 - 1$ |
- 21.** By first taking out the highest common factor, factorise fully each of the following.
(The first one is done for you.)
- (i) $3a^2 - 12b^2 = 3(a^2 - 4b^2) = 3(a + 2b)(a - 2b)$
- (ii) $5c^2 - 5d^2$ (iii) $3a^2 - 27b^2$ (iv) $3x^2 - 75y^2$ (v) $12a^2 - 27$

Use factors to find the value of each of these:

- | | | | |
|--------------------------|--------------------------|---------------------------|--------------------------|
| 22. $27^2 - 3^2$ | 23. $41^2 - 39^2$ | 24. $101^2 - 99^2$ | 25. $55^2 - 54^2$ |
| 26. $62^2 - 52^2$ | 27. $76^2 - 24^2$ | 28. $190^2 - 10^2$ | 29. $293^2 - 7^2$ |

HL DIFFERENCE OF TWO SQUARES

Exercise 2.3

Factorise each of the following:

1. $x^2 - y^2$
 2. $a^2 - b^2$
 3. $x^2 - 4y^2$
 4. $x^2 - 16y^2$
 5. $4x^2 - y^2$
 6. $9x^2 - 16y^2$
 7. $4a^2 - 25b^2$
 8. $36x^2 - 49y^2$
 9. $64x^2 - 9y^2$
 10. $16x^2 - 25$
 11. $25x^2 - 1$
 12. $36x^2 - 25$
 13. $15x^2 - 64y^2$
 14. $1 - 36x^2$
 15. $1 - 81y^2$
 16. $36 - 121y^2$
 17. $49a^2 - 4b^2$
 18. $(xy)^2 - 4$
 19. $(ab)^2 - 25$
 20. $x^2y^2 - 16$
 21. $a^2b^2 - 49$
 22. $(5xy)^2 - 36$
 23. $16a^2b^2 - 25$
 24. $9x^2y^2 - 1$
 25. $4a^2b^2 - 49c^2d^2$
 26. $121a^2 - 64b^2c^2$
 27. $81h^2k^2 - 25p^2q^2$
28. First take out the highest common factor and then factorise each of the following:
- | | | |
|--------------------|---------------------|------------------------|
| (i) $3x^2 - 27y^2$ | (ii) $12x^2 - 3y^2$ | (iii) $27x^2 - 3y^2$ |
| (iv) $45 - 5x^2$ | (v) $45k^2 - 20$ | (vi) $4a^2x^2 - 36y^2$ |
29. **Challenge:** factorise $a^4 - b^4$ as far as possible.
30. Use the difference of two squares to evaluate each of these:
- | | | | |
|------------------|--------------------|---------------------------|--------------------------|
| (i) $96^2 - 4^2$ | (ii) $23^2 - 17^2$ | (iii) $(7.9)^2 - (2.1)^2$ | (iv) $(9.4)^2 - (0.6)^2$ |
|------------------|--------------------|---------------------------|--------------------------|
31. Simplify $(3x + b)(6x - 2b) - (2y + b)(4y - 2b)$.
Now factorise fully the simplified expression.
32. Simplify and hence factorise $(3x - 2y)^2 - y(5y - 12x)$.

OL: Factorising Quadratics

Section 4.4 Factorising quadratic expressions

An expression of the form $ax^2 + bx + c$ – where a , b and c are numbers – is called a **quadratic expression** since the highest power of x is 2.

Since $(x + 5)(x + 2) = x^2 + 7x + 10$, we say that $(x + 5)$ and $(x + 2)$ are the factors of $x^2 + 7x + 10$.

In the product $(x + 5)(x + 2) = x^2 + 7x + 10$,

outside terms
 $(x + 5)(x + 2)$
inside terms

- (i) x^2 is obtained from the product $x \times x$
- (ii) 10 is the product of 5 and 2, the two number terms
- (iii) $7x$ is obtained by adding the product of the outside terms to the product of the inside terms,
i.e. $2x + 5x = 7x$.

An expression in the form $ax^2 + bx + c$ is generally called a quadratic trinomial as it contains 3 terms.

We factorise a quadratic expression by 'trial and error' to find numbers such that the product of the outside terms, added to the product of the inside terms, gives the middle term.

Here are the factors of $x^2 + 7x + 12$:

$$\begin{array}{c} \text{outside terms} \\ \text{---} \\ (x + 4)(x + 3) \\ \text{---} \\ \text{inside terms} \end{array}$$

$$\begin{array}{l} 3x + 4x = 7x \\ 3 \times 4 = 12 \end{array}$$

Example 1

Factorise $x^2 + 8x + 15$.

The factors of $x^2 + 8x + 15$ will take the form $(x + ?)(x + ?)$

We are looking for the factors of 15 whose sum is 8.

These factors are 3 and 5.

\therefore the factors are $(x + 3)(x + 5)$.

Note: To verify that your factors are correct, use the middle-term check as follows:

$$\begin{array}{c} \text{outside terms} \\ \text{---} \\ (x + 3)(x + 5) \\ \text{---} \\ \text{inside terms} \end{array} \quad \text{i.e. } 5x + 3x = 8x \dots \text{correct}$$

Third term positive, middle term negative

If the third term of a quadratic expression is positive and the middle term is negative, e.g. $x^2 - 8x + 15$, then the factors will take the form

$$(x - ?)(x - ?)$$

Example 2

Find the factors of $x^2 - 11x + 28$.

The factors of 28 which add up to -11 are -7 and -4 .

\therefore the factors are $(x - 7)(x - 4)$.

$$\text{Verification: } \begin{array}{c} -4x \\ \text{---} \\ (x - 7)(x - 4) \\ \text{---} \\ -7x \end{array} \quad \text{i.e. } -4x - 7x = -11x.$$

Exercise 4.4

Find the factors of each of the following:

- | | | |
|----------------------|----------------------|----------------------|
| 1. $x^2 + 3x + 2$ | 2. $x^2 + 4x + 4$ | 3. $x^2 + 8x + 7$ |
| 4. $x^2 + 5x + 6$ | 5. $x^2 + 8x + 12$ | 6. $x^2 + 9x + 14$ |
| 7. $a^2 + 7a + 12$ | 8. $x^2 + 4x + 3$ | 9. $a^2 + 6a + 8$ |
| 10. $a^2 + 9a + 18$ | 11. $a^2 + 10a + 16$ | 12. $a^2 + 10a + 24$ |
| 13. $x^2 + 11x + 24$ | 14. $x^2 + 12x + 20$ | 15. $x^2 + 12x + 27$ |
| 16. $x^2 + 13x + 22$ | 17. $x^2 + 13x + 30$ | 18. $x^2 + 11x + 30$ |
| 19. $x^2 + 14x + 33$ | 20. $x^2 + 15x + 36$ | 21. $x^2 + 15x + 44$ |
| 22. $x^2 + 9x + 20$ | 23. $x^2 + 18x + 17$ | 24. $x^2 + 18x + 32$ |
| 25. $x^2 - 4x + 3$ | 26. $x^2 - 5x + 6$ | 27. $x^2 - 6x + 8$ |
| 28. $x^2 - 5x + 4$ | 29. $x^2 - 7x + 12$ | 30. $x^2 - 7x + 10$ |
| 31. $x^2 - 9x + 14$ | 32. $x^2 - 11x + 24$ | 33. $x^2 - 8x + 15$ |
| 34. $x^2 - 9x + 18$ | 35. $x^2 - 9x + 20$ | 36. $x^2 - 12x + 20$ |
| 37. $x^2 - 12x + 35$ | 38. $x^2 - 10x + 24$ | 39. $x^2 - 14x + 24$ |
| 40. $x^2 - 12x + 27$ | 41. $x^2 - 11x + 30$ | 42. $x^2 - 13x + 30$ |
| 43. $x^2 - 17x + 30$ | 44. $x^2 - 13x + 36$ | 45. $x^2 - 15x + 36$ |
| 46. $x^2 - 14x + 45$ | 47. $x^2 - 14x + 40$ | 48. $x^2 - 13x + 40$ |

Section 4.5 Quadratic expressions – Third term negative —Take the expression $x^2 - 2x - 8$.

Here the final term is negative.

When the final term is negative, the factors will be of the form

$$(x + ?)(x - ?) \dots \text{[or } (x - ?)(x + ?)].$$

In this case, the factors are $(x + 2)(x - 4)$.

Check: $(x + 2)(x - 4) \dots -4x + 2x = -2x \dots \text{correct}$

Exercise 4.5

Factorise each of the following:

1. $x^2 - x - 2$

2. $x^2 - 2x - 3$

3. $x^2 - 3x - 4$

4. $x^2 + 2x - 3$

5. $x^2 - x - 12$

6. $x^2 + x - 12$

7. $x^2 - 4x - 12$

8. $x^2 - 3x - 10$

9. $x^2 - 9x - 10$

10. $x^2 - 5x - 14$

11. $x^2 - 2x - 15$

12. $x^2 + 7x - 18$

13. $x^2 + 3x - 18$

14. $x^2 + 4x - 21$

15. $x^2 - 2x - 24$

16. $x^2 + 5x - 24$

17. $x^2 - 10x - 24$

18. $x^2 - x - 30$

19. $x^2 + 7x - 30$

20. $x^2 - 13x - 30$

21. $x^2 + 3x - 28$

22. $x^2 - 12x - 28$

23. $x^2 + 5x - 36$

24. $x^2 - 9x - 36$

25. $x^2 + 6x - 40$

26. $x^2 - 3x - 40$

27. $x^2 - 18x - 40$

28. $x^2 - 4x - 45$

29. $x^2 + 6x - 16$

30. $x^2 - 8x - 48$

Factorise each of the following which contain examples of the three types we have met so far:

31. $x^2 + 9x + 14$

32. $x^2 - 12x + 27$

33. $x^2 + 2x - 24$

34. $x^2 - 8x + 7$

35. $x^2 - 9x + 14$

36. $x^2 - 4x - 32$

37. $x^2 + 14x + 24$

38. $x^2 + x - 42$

39. $x^2 + 5x - 50$

40. $x^2 - 11x + 28$

41. $x^2 - 4x - 60$

42. $x^2 - 17x + 60$

HL: Factorising Quadratics

We factorise a quadratic expression by 'trial and error' to find numbers such that the product of the outside terms added to the product of the inside terms gives the middle term.

Here are the factors of $x^2 + 8x + 15$:

$$\begin{array}{c} \text{outside terms} \\ \text{---} \\ (x + 5)(x + 3) \\ \text{---} \\ \text{inside terms} \end{array}$$

$$\begin{array}{l} 3x + 5x = 8x \\ 3 \times 5 = 15 \end{array}$$

Example 1

Factorise $3x^2 + 10x + 8$.

The factors of $3x^2 + 10x + 8$ will take the form $(3x + ?)(x + ?)$

The factors of 8 are 8×1 or 4×2

Investigate 4 and 2: $(3x + 2)(x + 4)$

$$\begin{array}{l} \text{Outside + inside terms} \\ = 12x + 2x \\ = 14x \dots \text{incorrect} \end{array}$$

Try again: $(3x + 4)(x + 2)$

$$\begin{array}{l} \text{Outside + inside terms} \\ = 6x + 4x = 10x \\ \dots \text{correct} \end{array}$$

The factors of $3x^2 + 10x + 8$ are $(3x + 4)(x + 2)$

Final term positive

If the third term of a quadratic expression is positive and the middle term is negative, e.g. $x^2 - 8x + 15$, the factors will take the form shown on the right.

$$(x - ?)(x - ?)$$

Example 2

Find the factors of $2x^2 - 11x + 12$.

The factors will take the form $(2x - ?)(x - ?)$

$$\begin{array}{c} -8x \\ \text{---} \\ (2x - 3)(x - 4) \dots \text{correct factors} \\ \text{---} \\ -3x \end{array}$$

$$\begin{array}{l} \text{Factors of 12:} \\ 6 \times 2 \\ 4 \times 3 \\ 12 \times 1 \end{array}$$

$$\therefore 2x^2 - 11x + 12 = (2x - 3)(x - 4)$$

Final term negative

If the final term is negative, the factors will take either of the forms shown on the right.

$$\begin{array}{c} (x + ?)(x - ?) \\ \text{or} \\ (x - ?)(x + ?) \end{array}$$

Example 3

Factorise (i) $8x^2 + 10x - 3$ (ii) $7x^2 - 19xy - 6y^2$

$$(i) \quad 8x^2 + 10x - 3 = (4x - 1)(2x + 3)$$
$$12x - 2x = 10x \quad (\text{correct})$$

$$\therefore 8x^2 + 10x - 3 = (4x - 1)(2x + 3)$$

$$(ii) \quad 7x^2 - 19xy - 6y^2 = (7x + 3y)(x - 2y)$$
$$-14xy + 3xy = -11xy \quad (\text{incorrect})$$

$$(7x + 2y)(x - 3y)$$
$$-21xy + 2xy = -19xy \quad (\text{correct})$$

$$\therefore 7x^2 - 19xy - 6y^2 = (7x + 2y)(x - 3y)$$

Exercise 2.4

Factorise each of the following:

1. $x^2 + 5x + 6$
2. $x^2 + 8x + 12$
3. $x^2 + 9x + 14$
4. $x^2 + 11x + 24$
5. $x^2 + 12x + 20$
6. $x^2 + 12x + 27$
7. $x^2 + 11x + 30$
8. $x^2 + 15x + 44$
9. $x^2 + 20x + 36$
10. $2x^2 + 5x + 2$
11. $2x^2 + 11x + 14$
12. $5x^2 + 21x + 4$
13. $x^2 - 7x + 12$
14. $x^2 - 9x + 18$
15. $x^2 - 9x + 20$
16. $x^2 - 14x + 24$
17. $x^2 - 12x + 27$
18. $x^2 - 13x + 36$
19. $2x^2 - 7x + 3$
20. $3x^2 - 17x + 10$
21. $5x^2 - 17x + 6$
22. $3x^2 - 17x + 20$
23. $5x^2 + 27x - 18$
24. $3x^2 - 14x + 15$
25. $x^2 - 4x - 12$
26. $x^2 - 3x - 10$
27. $x^2 + 7x - 18$
28. $x^2 + 7x - 30$
29. $x^2 - 13x - 30$
30. $x^2 - 18x - 40$
31. $2x^2 - 7x - 15$
32. $3x^2 + 11x - 20$
33. $5x^2 - 12x - 9$
34. $x^2 - 6x - 72$
35. $8x^2 + 10x - 3$
36. $2x^2 - 19x + 9$
37. $12x^2 - 11x - 5$
38. $6x^2 + x - 15$
39. $8x^2 - 14x + 3$
40. $3x^2 + 13x - 10$
41. $9x^2 + 24x + 16$
42. $5x^2 - 31x + 6$
43. $3x^2 - x - 14$
44. $6x^2 - 11x + 3$
45. $12x^2 - 23x + 10$
46. $9x^2 + 25x - 6$
47. $6x^2 + x - 22$
48. $9x^2 - x - 10$
49. $4x^2 - 11x + 6$
50. $10x^2 - 17x - 20$
51. $36x^2 - 7x - 4$
52. $12x^2 - 17x + 6$
53. $15x^2 - 14x - 8$
54. $24x^2 + 2x - 15$

HL: Algebraic Fractions

Section 2.5 Using factors to simplify algebraic fractions

The fraction $\frac{10}{15}$ can be simplified by dividing above and below by the common factor 5. $\frac{\cancel{2}10}{\cancel{3}15} = \frac{2}{3}$

Similarly, $\frac{x^2 - 4}{x + 2}$ can be simplified by dividing above and below by a common factor.

$$\frac{x^2 - 4}{x + 2} = \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}} = x - 2$$

Example 1

Simplify (i) $\frac{3n - 12}{n - 4}$

(ii) $\frac{3x^2 - 5x - 2}{x - 2}$

$$(i) \frac{3n - 12}{n - 4} = \frac{3\cancel{(n-4)}}{\cancel{(n-4)}} = 3$$

$$(ii) \frac{3x^2 - 5x - 2}{x - 2} = \frac{(3x + 1)\cancel{(x - 2)}}{\cancel{(x - 2)}} = 3x + 1$$

Exercise 2.5

1. Simplify each of the following:

(i) $\frac{14}{35}$

(ii) $\frac{7x}{14}$

(iii) $\frac{9x^2}{3x}$

(iv) $\frac{8p^2}{2p}$

(v) $\frac{9x^2y}{3xy}$

2. Simplify each of the following:

(i) $\frac{4x + 4y}{4}$

(ii) $\frac{12(a + b)}{3(a + b)}$

(iii) $\frac{3x + 12}{x(x + 4)}$

(iv) $\frac{4a - 8b}{3(a - 2b)}$

Simplify each of the following, using factors where necessary:

3. $\frac{(x - 1)(x + 3)}{x + 3}$

4. $\frac{2(y - 1)(y + 3)}{y - 1}$

5. $\frac{x^2 + 8x + 7}{x + 1}$

6. $\frac{x - 4}{x^2 - 6x + 8}$

7. $\frac{x - 2}{x^2 + 5x - 14}$

8. $\frac{3x - 3}{x^2 - 2x + 1}$

9. $\frac{2x - 6}{x^2 + x - 12}$

10. $\frac{x^2 + x - 30}{x - 5}$

11. $\frac{a^2 + 2ab}{3a + 6b}$

12. $\frac{x^2 - 9}{x - 3}$

13. $\frac{a^2 - 16}{3a - 12}$

14. $\frac{n + 9}{n^2 + 18n + 81}$

15. $\frac{4x - 8}{x^2 - 4}$

16. $\frac{2x^2 + 5x - 3}{2x - 1}$

17. $\frac{2x^2 + 11x + 15}{2x + 5}$

18. $\frac{ab - ac}{b - c}$

19. $\frac{5 - x}{x - 5}$

20. $\frac{3a + 9}{a^2 - 1} \div \frac{a + 3}{a - 1}$

Text & Tests 2 Ordinary Level

- | | |
|-----------------------|-----------------------|
| 21. $(x + 11)(x + 4)$ | 22. $(x + 5)(x + 4)$ |
| 23. $(x + 17)(x + 1)$ | 24. $(x + 16)(x + 2)$ |
| 25. $(x - 3)(x - 1)$ | 26. $(x - 3)(x - 2)$ |
| 27. $(x - 4)(x - 2)$ | 28. $(x - 4)(x - 1)$ |
| 29. $(x - 4)(x - 3)$ | 30. $(x - 5)(x - 2)$ |
| 31. $(x - 7)(x - 2)$ | 32. $(x - 8)(x - 3)$ |
| 33. $(x - 5)(x - 3)$ | 34. $(x - 6)(x - 3)$ |
| 35. $(x - 5)(x - 4)$ | 36. $(x - 10)(x - 2)$ |
| 37. $(x - 7)(x - 5)$ | 38. $(x - 6)(x - 4)$ |
| 39. $(x - 12)(x - 2)$ | 40. $(x - 9)(x - 3)$ |
| 41. $(x - 6)(x - 5)$ | 42. $(x - 10)(x - 3)$ |
| 43. $(x - 15)(x - 2)$ | 44. $(x - 9)(x - 4)$ |
| 45. $(x - 12)(x - 3)$ | 46. $(x - 9)(x - 5)$ |
| 47. $(x - 10)(x - 4)$ | 48. $(x - 8)(x - 5)$ |

Exercise 4.5

- | | |
|-----------------------|-----------------------|
| 1. $(x - 2)(x + 1)$ | 2. $(x - 3)(x + 1)$ |
| 3. $(x - 4)(x + 1)$ | 4. $(x - 1)(x + 3)$ |
| 5. $(x - 4)(x + 3)$ | 6. $(x - 3)(x + 4)$ |
| 7. $(x - 6)(x + 2)$ | 8. $(x - 5)(x + 2)$ |
| 9. $(x - 10)(x + 1)$ | 10. $(x - 7)(x + 2)$ |
| 11. $(x - 5)(x + 3)$ | 12. $(x - 2)(x + 9)$ |
| 13. $(x - 3)(x + 6)$ | 14. $(x - 3)(x + 7)$ |
| 15. $(x - 6)(x + 4)$ | 16. $(x - 3)(x + 8)$ |
| 17. $(x - 12)(x + 2)$ | 18. $(x - 6)(x + 5)$ |
| 19. $(x - 3)(x + 10)$ | 20. $(x - 15)(x + 2)$ |
| 21. $(x - 4)(x + 7)$ | 22. $(x - 14)(x + 2)$ |
| 23. $(x - 4)(x + 9)$ | 24. $(x + 3)(x - 12)$ |
| 25. $(x - 4)(x + 10)$ | 26. $(x - 8)(x + 5)$ |
| 27. $(x - 20)(x + 2)$ | 28. $(x - 9)(x + 5)$ |
| 29. $(x - 2)(x + 8)$ | 30. $(x + 4)(x - 12)$ |
| 31. $(x + 7)(x + 2)$ | 32. $(x - 9)(x - 3)$ |
| 33. $(x - 4)(x + 6)$ | 34. $(x - 7)(x - 1)$ |
| 35. $(x - 7)(x - 2)$ | 36. $(x - 8)(x + 4)$ |
| 37. $(x + 12)(x + 2)$ | 38. $(x - 6)(x + 7)$ |
| 39. $(x - 5)(x + 10)$ | 40. $(x - 7)(x - 4)$ |
| 41. $(x - 10)(x + 6)$ | 42. $(x - 12)(x - 5)$ |

Test yourself 4

- | | |
|---------------------------|---------------------------|
| (a) (i) $9x(x + 4)$ | (ii) $5b(a + 3c)$ |
| (iii) $6a(x - 2y)$ | (iv) $6x(x^2 - 3y)$ |
| (b) (i) $(a + b)(7 + x)$ | (ii) $(a - 5b)(a + c)$ |
| (c) (i) $(x - 5)(x - 11)$ | (ii) $(4x + 11)(4x - 11)$ |
- | | |
|------------------------------|-----------------|
| (a) (i) $(p - 2)$ | (ii) $(4n + 1)$ |
| (iii) $(3x - 1)$ | (iv) $(d - 3)$ |
| (b) (i) $(n + 1)$ | (ii) $(n + 4)$ |
| (iii) $(n + 6)$ | (iv) $(n - 8)$ |
| (c) (i) $(12 + 3b)(12 - 3b)$ | |
| (ii) 600 | |
- | | |
|----------------------------|-----------------------|
| (a) (i) $(x - 3z)$ | (ii) $(x - 3z)$ |
| (b) (i) $(2a - b)(3x + y)$ | (ii) $(x - 6)(x - 9)$ |
| (c) (i) $(6x + 1)(6x - 1)$ | (ii) 1080 |

Answers HL: Factors

Chapter 2: Factors

Exercise 2.1

- | | |
|------------------------|------------------------|
| 1. (i) 3 | (ii) 6 |
| (iii) 7 | (iv) 7 |
| 2. (i) $4x$ | (ii) $3n$ |
| (iii) $5x$ | (iv) $3a$ |
| (v) $3x$ | (vi) $2ab$ |
| 3. (i) $(x + 2y)$ | (ii) $(2a + 3b)$ |
| (iii) $(a + c)$ | (iv) $(a + 2)$ |
| (v) $(x - 3y)$ | (vi) $(2x - 3z)$ |
| (vii) $(3x + 2y)$ | (viii) $(3a - 4b + 2)$ |
| 4. $6(x + 3y)$ | 5. $3b(a + c)$ |
| 6. $6a(x - 2y)$ | 7. $6a(a - 2)$ |
| 8. $7x(x - 4)$ | 9. $5x(3x + 5y)$ |
| 10. $3x^2(1 - 2y)$ | 11. $3ab(b - 2)$ |
| 12. $3p(p - 2q)$ | 13. $2x^2(y - 3z)$ |
| 14. $2y^2(3z + 5)$ | 15. $5pq(2p + q)$ |
| 16. $2a(a^2 - 2a + 4)$ | 17. $2x(2x - 3y + 4z)$ |

18. $5xy(y - 4x)$
 20. $5x(x^2 - 2x + 3)$
 22. (i) PARROT
 (iii) GROUSE
19. $4xy(xy - 2)$
 21. $2ab(a - 2b + 6c)$
 (ii) THRUSH
 (iv) STARLING

Exercise 2.2

1. $(2a + 3)(x + y)$
 3. $(3a - 4)(2b - c)$
 5. $(2a - 1)(x - 2y)$
 7. $(x + 3)(x - a)$
 9. $(a + 5)(b + 3)$
 11. $(c - 2d)(2c + 1)$
 13. $(2a + b)(c - 2d)$
 15. $(4a - 3b)(2x + y)$
 17. $(2y - z)(x - 1)$
 19. $(2x - 3)(xy - z)$
 21. $(2a + b)(2ab - 3)$
 23. $(2b - c)(5a - d)$
 25. $(2a - 3x)(3a - 5y)$
 27. $(ax - y)(3bx - 5y)$
 29. $(x - 2a)(x + b)$
2. $(3x - 4)(2a - b)$
 4. $(2x + b)(5y - z)$
 6. $(a + c)(a + b)$
 8. $(a - 5)(b + c)$
 10. $(3x + 4y)(x - z)$
 12. $(2a - 3)(x - 3y)$
 14. $(3xy + 2z)(1 - z)$
 16. $(3a - 4)(2x^2 + 3)$
 18. $(n - 5)(a + b)$
 20. $(7y + 2a)(y - 3b)$
 22. $(3a - 2b)(4a + 3c)$
 24. $(2x - a)(2x - 3y)$
 26. $(3y - 4z)(2x - 3y)$
 28. $(3a + 2c)(3a^2 - 2b)$
 30. $(2x - 3y)(3x - 2a)$

Exercise 2.3

1. $(x + y)(x - y)$
 3. $(x + 2y)(x - 2y)$
 5. $(2x + y)(2x - y)$
 7. $(2a + 5b)(2a - 5b)$
 9. $(8x + 3y)(8x - 3y)$
 11. $(5x + 1)(5x - 1)$
 13. $(5x + 8y)(5x - 8y)$
 15. $(1 + 9y)(1 - 9y)$
 17. $(7a + 2b)(7a - 2b)$
 19. $(ab + 5)(ab - 5)$
 21. $(ab + 7)(ab - 7)$
 23. $(4ab + 5)(4ab - 5)$
 25. $(2ab + 7cd)(2ab - 7cd)$
 26. $(11a + 8bc)(11a - 8bc)$
 27. $(9hk + 5pq)(9hk - 5pq)$
 28. (i) $3(x + 3y)(x - 3y)$ (ii) $3(2x + y)(2x - y)$
 (iii) $3(3x + y)(3x - y)$ (iv) $5(3 + x)(3 - x)$
 (v) $5(3k + 2)(3k - 2)$
 (vi) $4(ax + 3y)(ax - 3y)$
 29. $(a^2 + b^2)(a + b)(a - b)$
 30. (i) 9200 (ii) 240 (iii) 88
 31. $2(9x^2 - 4y^2); 2(3x + 2y)(3x - 2y)$
 32. $(3x + y)(3x - y)$

Exercise 2.4

1. $(x + 2)(x + 3)$
 3. $(x + 7)(x + 2)$
 5. $(x + 2)(x + 10)$
 7. $(x + 5)(x + 6)$
 9. $(x + 2)(x + 18)$
2. $(x + 2)(x + 6)$
 4. $(x + 3)(x + 8)$
 6. $(x + 3)(x + 9)$
 8. $(x + 4)(x + 11)$
 10. $(2x + 1)(x + 2)$

11. $(2x + 7)(x + 2)$
 13. $(x - 3)(x - 4)$
 15. $(x - 4)(x - 5)$
 17. $(x - 3)(x - 9)$
 19. $(2x - 1)(x - 3)$
 21. $(5x - 2)(x - 3)$
 23. $(5x - 3)(x + 6)$
 25. $(x + 2)(x - 6)$
 27. $(x - 2)(x + 9)$
 29. $(x + 2)(x - 15)$
 31. $(2x + 3)(x - 5)$
 33. $(5x + 3)(x - 3)$
 35. $(4x - 1)(2x + 3)$
 37. $(4x - 5)(3x + 1)$
 39. $(4x - 1)(2x - 3)$
 41. $(3x + 4)(3x + 4)$
 43. $(3x - 7)(x + 2)$
 45. $(4x - 5)(3x - 2)$
 47. $(6x - 11)(x + 2)$
 49. $(4x - 3)(x - 2)$
 51. $(9x - 4)(4x + 1)$
 53. $(3x - 4)(5x + 2)$
12. $(5x + 1)(x + 4)$
 14. $(x - 3)(x - 6)$
 16. $(x - 2)(x - 12)$
 18. $(x - 4)(x - 9)$
 20. $(3x - 2)(x - 5)$
 22. $(3x - 5)(x - 4)$
 24. $(3x - 5)(x - 3)$
 26. $(x + 2)(x - 5)$
 28. $(x - 3)(x + 10)$
 30. $(x + 2)(x - 20)$
 32. $(3x - 4)(x + 5)$
 34. $(x + 6)(x - 12)$
 36. $(2x - 1)(x - 9)$
 38. $(3x + 5)(2x - 3)$
 40. $(3x - 2)(x + 5)$
 42. $(5x - 1)(x - 6)$
 44. $(3x - 1)(2x - 3)$
 46. $(9x - 2)(x + 3)$
 48. $(9x - 10)(x + 1)$
 50. $(5x + 4)(2x - 5)$
 52. $(4x - 3)(3x - 2)$
 54. $(6x + 5)(4x - 3)$

Exercise 2.5

1. (i) $\frac{2}{5}$ (ii) $\frac{x}{2}$ (iii) $3x$ (iv) $4p$ (v) $3x$
 2. (i) $x + y$ (ii) 4 (iii) $\frac{3}{x}$ (iv) $\frac{4}{3}$
 3. $x - 1$ 4. $2(y + 3)$ 5. $x + 7$
 6. $\frac{1}{x - 2}$ 7. $\frac{1}{x + 7}$ 8. $\frac{3}{x - 1}$
 9. $\frac{2}{x + 4}$ 10. $x + 6$ 11. $\frac{a}{3}$
 12. $x + 3$ 13. $\frac{a + 4}{3}$ 14. $\frac{1}{n + 9}$
 15. $\frac{4}{x + 2}$ 16. $x + 3$ 17. $x + 3$
 18. a 19. -1 20. $\frac{3}{a + 1}$

Test yourself 2

1. (a) (i) $(x - 3y)$ (ii) $(x - 4)(x - 6)$
 (b) (i) $(a + b)(7 + x)$ (ii) $(5a + 9)(5a - 9)$
 (c) $\frac{4x + 1}{4x}$
 2. (a) (i) $(3x - 2)(2x + 1)$
 (ii) $(3a + b)(2a + c)$
 (b) (i) $3ax(2a + x - 3)$
 (ii) $3(x + 4)(x - 4)$
 (c) $\frac{3x + 2}{2x + 5}$
 3. (a) (i) $2ab(4a + b)$ (ii) $(3x - 7)(x - 3)$
 (b) (i) $2(x + 2y)(x - 2y)$
 (ii) $(x - 1)(2y - z)$
 (c) $12(x + a)(x - a)$

Topic 3: Fractions

In this section you must be able to:

- 1. Simplifying fractions**
- 2. Solve equations involving fractions**

Algebraic Fractions – Formulae

chapter

21

Key words

algebraic fractions simplifying terms LCM (lowest common multiple)
mathematical sentence algebraic division rearranging formulae
changing the subject of a formula writing formulae

Section 21.1 Adding algebraic fractions

To add $\frac{3}{4} + \frac{4}{5}$, we find the LCM of 4 and 5, i.e. 20.

We now express each fraction with 20 as denominator.

$$\therefore \frac{3}{4} + \frac{4}{5} = \frac{5(3)}{20} + \frac{4(4)}{20} = \frac{15}{20} + \frac{16}{20} = \frac{15 + 16}{20} = \frac{31}{20}$$

$$\text{Similarly, } \frac{6}{7} - \frac{2}{3} = \frac{3(6) - 7(2)}{21} = \frac{18 - 14}{21} = \frac{4}{21}$$

Algebraic fractions can be added or subtracted in the same way as numerical fractions.

Example 1

Express as a single fraction: $\frac{5x - 3}{2} - \frac{2x + 1}{3}$

The LCM of 2 and 3 is 6.

$$\begin{aligned} \frac{5x - 3}{2} - \frac{2x + 1}{3} &= \frac{3(5x - 3) - 2(2x + 1)}{6} \\ &= \frac{15x - 9 - 4x - 2}{6} = \frac{11x - 11}{6} \end{aligned}$$

Simplifying algebraic terms

The fraction $\frac{8}{12}$ can be simplified by dividing the numerator and denominator by 4, as shown.

$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} = \frac{2}{3}$$

Similarly, the algebraic fraction $\frac{4ab}{2b}$ can be simplified by dividing the numerator and denominator by any common factors.

$$\text{Thus, } \frac{4ab}{2b} = \frac{\overset{2}{\cancel{4}} \times a \times \underset{1}{\cancel{b}}}{\underset{1}{\cancel{2}} \times b_1} = 2 \times a = 2a$$

Example 2

Simplify each of the following:

$$(i) \frac{9x^2y}{3x}$$

$$(ii) \frac{12a^3b}{ab^2}$$

$$(i) \frac{9x^2y}{3x} = \frac{\overset{3}{9} \times \overset{2}{x} \times x \times y}{\underset{1}{3} \times \underset{1}{x_1}}$$

$$= 3xy$$

$$(ii) \frac{12a^3b}{ab^2} = \frac{12 \times \overset{1}{a} \times \overset{3}{a} \times \overset{1}{b}}{\underset{1}{a} \times \underset{2}{b} \times \underset{1}{b_1}}$$

$$= \frac{12a^2}{b}$$

Exercise 21.1

1. Simplify each of the following expressions:

$$(i) \frac{10ab}{2b}$$

$$(ii) \frac{8xy}{4x}$$

$$(iii) \frac{15cd}{5d}$$

$$(iv) \frac{18ab}{6a}$$

$$(v) \frac{8x^2y}{4xy}$$

$$(vi) \frac{16b^2c}{2c}$$

$$(vii) \frac{14x^3y}{2x^2y}$$

$$(viii) \frac{28ab^2}{7ab}$$

2. Simplify these expressions:

$$(i) \frac{3}{x} \times \frac{4x}{9}$$

$$(ii) \frac{km}{4n} \times \frac{2n}{m}$$

$$(iii) \frac{ab}{3} \times \frac{6b}{a}$$

$$(iv) \frac{2ab \times 6a}{3a}$$

$$(v) \frac{x}{3} \div \frac{x}{6}$$

$$(vi) \frac{3}{2x} \div \frac{1}{3x}$$

$$(vii) \frac{3ab^2}{2} \div \frac{ab}{6}$$

$$(viii) \frac{8a \times 3ak}{2a \times 6k}$$

3.

A	E	G	L	M	N	O	P	R
2b	4bc	bc ²	4d	cd	2b ²	3bd	3c	2b ² d

Simplify each expression below as far as you can.

Use the code above to find a letter for each expression.

Rearrange each set of letters to spell a fruit.

$$(i) \frac{8cd}{2c}$$

$$\frac{12bc}{3}$$

$$\frac{15cd}{5d}$$

$$\frac{4bc}{2c}$$

$$\frac{9c^2b}{3cb}$$

$$(ii) \frac{18b^2d}{6b}$$

$$\frac{10ab^2}{5a}$$

$$\frac{20b^2c^2}{5bc}$$

$$\frac{12cd^5}{3cd^4}$$

$$\frac{5c^2d}{5c}$$

4. Express each of the following as a single fraction:

$$(i) \frac{1}{2} + \frac{2}{3}$$

$$(ii) \frac{3}{4} + \frac{3}{5}$$

$$(iii) \frac{5}{12} + \frac{2}{12} + \frac{1}{12}$$

$$(iv) \frac{7}{8} - \frac{3}{4}$$

$$(v) \frac{x}{2} + \frac{x}{3}$$

$$(vi) \frac{x}{5} + \frac{x}{3}$$

Express each of these as a single fraction.

5. $\frac{3x}{4} + \frac{5x}{2}$

7. $\frac{x}{4} - \frac{x}{6}$

9. $\frac{7x}{5} - \frac{x}{2}$

11. $\frac{x+1}{2} + \frac{x}{2}$

13. $\frac{x+4}{3} + \frac{2x+1}{3}$

15. $\frac{3x-2}{4} + \frac{5x+6}{4}$

17. $\frac{2x+1}{3} + \frac{x+4}{2}$

19. $\frac{2x-1}{6} + \frac{x-3}{4}$

21. $\frac{2x-3}{2} + \frac{x-1}{4} - \frac{5}{6}$

23. $\frac{3x-1}{4} - \frac{x}{10} - \frac{4x+2}{5}$

25. $\frac{3}{5} - \frac{2x-1}{10} + \frac{3x-2}{4}$

6. $\frac{3x}{5} + \frac{2x}{3}$

8. $\frac{x}{2} - \frac{x}{5}$

10. $\frac{3x}{4} - \frac{2x}{5}$

12. $\frac{x+1}{2} + \frac{x+4}{2}$

14. $\frac{3x-4}{5} + \frac{5x-2}{5}$

16. $\frac{2x+1}{3} + \frac{2x-3}{3}$

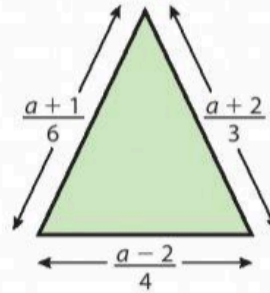
18. $\frac{2x-1}{3} + \frac{x+2}{4}$

20. $\frac{5x-1}{4} - \frac{2x-4}{5}$

22. $\frac{x-6}{3} + \frac{3}{4} - \frac{3x-4}{2}$

24. $\frac{3x+5}{6} - \frac{1}{12} - \frac{2x+3}{4}$

26. Express in its simplest form the perimeter of the given triangle.



Section 21.2 Solving equations involving fractions

Consider the equation $\frac{x-1}{5} = 4$.

To get rid of the fraction, we multiply both sides by 5.

$$\begin{aligned} \therefore \frac{x-1}{5} &= 4 & \text{5} \\ x-1 &= 20 \\ x-1+1 &= 20+1 \\ x &= 21 \end{aligned}$$

If an equation contains more than one fraction, we multiply each part by the lowest common multiple (LCM) of the denominators.

Example 1

Solve each of these equations:

(i) $\frac{3x}{4} - \frac{x}{2} = 3$

- (ii) The LCM of 4 and 2 is 4.
Multiply each term by 4.

$$\frac{4(3x)}{4} - \frac{4(x)}{2} = 3 \quad 4$$
$$3x - 2x = 12$$
$$x = 12$$

(ii) $\frac{2x - 5}{3} = \frac{x - 2}{2}$

- (ii) The LCM of 3 and 2 is 6.
Multiply each term by 6.

$$\frac{6(2x - 5)}{3} = \frac{6(x - 2)}{2}$$
$$2(2x - 5) = 3(x - 2)$$
$$4x - 10 = 3x - 6$$
$$4x - 3x - 10 = 3x - 6 - 3x$$
$$x - 10 = -6$$
$$x - 10 + 10 = -6 + 10$$
$$x = 4$$

Example 2

Solve the equation: $\frac{3x - 1}{6} - \frac{x - 3}{4} = \frac{4}{3}$

The LCM of 6, 4 and 3 is 12.

We now multiply each term by 12.

$$\frac{12(3x - 1)}{6} - \frac{12(x - 3)}{4} = \frac{12(4)}{3}$$
$$2(3x - 1) - 3(x - 3) = 4(4)$$
$$6x - 2 - 3x + 9 = 16$$
$$3x + 7 = 16$$
$$3x + 7 - 7 = 16 - 7$$
$$3x = 9$$
$$x = 3$$

Exercise 21.2

Solve the following equations:

1. $\frac{2x}{3} = 6$

2. $\frac{3x}{5} = 3$

3. $\frac{x}{2} = \frac{6}{4}$

4. $\frac{3x}{4} = \frac{9}{2}$

5. $\frac{5x}{3} = 10$

6. $\frac{x - 3}{2} = 4$

7. $\frac{3x - 1}{4} = 2$

8. $\frac{3x - 1}{4} = 8$

9. $\frac{2x + 1}{3} = \frac{1}{2}$

10. $\frac{2x - 5}{3} = \frac{x - 2}{3}$

11. $\frac{x - 3}{4} = \frac{x - 2}{5}$

12. $\frac{x + 2}{6} = \frac{2x - 5}{3}$

13. $\frac{2x}{5} + \frac{x}{2} = \frac{9}{2}$

16. $\frac{2x}{3} - \frac{x}{4} = \frac{5}{2}$

19. $\frac{x+2}{4} + \frac{x-3}{2} = \frac{1}{2}$

22. $\frac{x-2}{5} + \frac{2x-3}{10} = \frac{1}{2}$

25. $\frac{x-2}{3} + \frac{x-3}{4} = \frac{x-1}{2}$

28. $\frac{3(x-4)}{5} + 3 = \frac{3(x-5)}{2}$

14. $\frac{2x}{3} - \frac{x}{4} = \frac{5}{6}$

17. $\frac{2x-1}{3} + \frac{x}{4} = \frac{6}{4}$

20. $\frac{x+2}{4} - \frac{x-3}{3} = \frac{1}{2}$

23. $\frac{2x-3}{5} + \frac{1}{20} = \frac{x-1}{4}$

26. $\frac{2x-1}{4} - \frac{x-1}{5} = 1$

29. $\frac{4x+5}{5} + \frac{2x+7}{2} = \frac{3x}{10}$

15. $\frac{3x}{4} - \frac{5x}{8} = \frac{1}{2}$

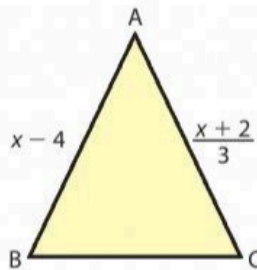
18. $\frac{x-3}{6} = \frac{x}{5} - \frac{3}{2}$

21. $\frac{3x-1}{6} - \frac{x-3}{4} = \frac{4}{3}$

24. $\frac{3x-1}{2} - \frac{2x-5}{3} = 2$

27. $\frac{3x-2}{6} - \frac{3x+1}{4} = \frac{2}{3}$

30. The diagram shows an isosceles triangle ABC.



If $|AB| = |AC|$,

- (i) write down an equation in terms of x .
- (ii) work out the length of $[AB]$.

Answers

Chapter 21: Algebraic Fractions – Formulae

Exercise 21.1

1. (i) $5a$ (ii) $2y$ (iii) $3c$ (iv) $3b$
(v) $2x$ (vi) $8b^2$ (vii) $7x$ (viii) $4b$
2. (i) $1\frac{1}{3}$ (ii) $\frac{k}{2}$ (iii) $2b^2$ (iv) $4ab$
(v) 2 (vi) $4\frac{1}{2}$ (vii) $9b$ (viii) $2a$
3. (i) APPLE (ii) LEMON or (MELON)
4. (i) $\frac{7}{6}$ (ii) $\frac{27}{20}$ (iii) $\frac{2}{3}$
(iv) $\frac{1}{8}$ (v) $\frac{5x}{6}$ (vi) $\frac{8x}{15}$
5. $\frac{13x}{4}$ 6. $\frac{19x}{15}$ 7. $\frac{x}{12}$ 8. $\frac{3x}{10}$
9. $\frac{9x}{10}$ 10. $\frac{7x}{20}$ 11. $\frac{2x+1}{2}$ 12. $\frac{2x+5}{2}$
13. $\frac{3x+5}{3}$ 14. $\frac{8x-6}{5}$ 15. $\frac{8x+4}{4}$
16. $\frac{4x-2}{3}$ 17. $\frac{7x+14}{6}$
18. $\frac{11x+2}{12}$ 19. $\frac{7x-11}{12}$
20. $\frac{17x+11}{20}$ 21. $\frac{15x-31}{12}$
22. $\frac{9-14x}{12}$ 23. $\frac{-3x-13}{20}$
24. 0 25. $\frac{11x+4}{20}$ 26. $\frac{9a+4}{12}$

Exercise 21.2

1. $x = 9$ 2. $x = 5$ 3. $x = 3$
4. 6 5. 6 6. 11 7. 3 8. 11
9. $\frac{1}{4}$ 10. 3 11. 7 12. 4 13. 5
14. 2 15. 4 16. 6 17. 2 18. 30
19. 2 20. 12 21. 3 22. 3 23. 2
24. 1 25. 11 26. $3\frac{1}{2}$ 27. -5 28. 9
29. -3 30. (i) $x - 4 = \frac{x+2}{3}$ (ii) $|AB| = 3$

Topic 4: Applied Arithmetic

In this section you must be able to solve problems incorporating:

1. VAT

2. Income Tax

Section 5.2 Applying percentages

When dealing with problems involving percentages, it is very important to be able to convert a percentage to a decimal.

When a percentage is expressed as a decimal, the electronic calculator is particularly useful for finding the required percentage.

Some examples are highlighted below:

To find 4%, multiply by 0.04.

To find 104%, multiply by 1.04.

To find $6\frac{1}{2}\%$, multiply by 0.065.

To find $106\frac{1}{2}\%$, multiply by 1.065.

To find 12%, multiply by 0.12.

To find 112%, multiply by 1.12.

To find 87%, multiply by 0.87.

To find 187%, multiply by 1.87.

1. Value-added tax (VAT)

Value-added tax or **VAT** is a government tax which is added to many of the things that we buy. In most shops, the marked price of an item includes VAT and so we do not have to calculate it.

Sometimes, the prices of more expensive items such as television sets, commercial vehicles and furniture are given without VAT. In these cases, the shopkeeper calculates the VAT amount and adds it to the price of the item.

If an article is priced at €280 plus VAT at 23%, the full price can be worked out as follows:

$$1\% \text{ of } \text{€}280 = \text{€}2.80$$

$$23\% \text{ of } \text{€}280 = \text{€}2.80 \times 23 = \text{€}64.40$$

$$\begin{aligned} \therefore \text{ the full price} &= \text{€}280 + \text{€}64.40 \\ &= \text{€}344.40 \end{aligned}$$



Using a calculator:

$$100\% + 23\% = 123\%$$

To find 123%, we multiply by 1.23

$$\begin{aligned} \text{Full price} &= \text{€}280 \times 1.23 \\ &= \text{€}344.40 \end{aligned}$$

Example 1

The rate of VAT on bicycles is 23%.

- Find the selling price of a bicycle priced at €360 + VAT.
- Another bicycle is advertised at €615 which includes VAT. Find the price of this bicycle before VAT was added.



(i) $23\% \text{ of } €360 = €360 \times 0.23 = €82.80$
 $\therefore \text{ the selling price of the bicycle } = €360 + €82.80$
 $= €442.80$

Or

Adding 23% gives 123%

$$123\% \text{ of } €360 = €360 \times 1.23 = €442.80 \quad \dots \text{ as above.}$$

(ii) €615 represents 123% of the price before VAT is added.

$$123\% = €615$$

$$1\% = € \frac{615}{123}$$

$$100\% = € \frac{615}{123} \times \frac{100}{1}$$

$$100\% = €500$$

\therefore the price of the bicycle before VAT was added was €500.

2. Percentage profit or loss

When dealing with percentage profit or loss, we base this percentage on the **cost price**, unless otherwise stated.

$$\text{Percentage profit} = \frac{\text{Profit}}{\text{Cost price}} \times \frac{100}{1} \% ; \quad \text{Percentage loss} = \frac{\text{Loss}}{\text{Cost price}} \times \frac{100}{1} \%$$

Example 3

- (i) A hardware store buys lawnmowers for €480 and sells them for €576. Calculate the percentage profit the store makes on the lawnmowers.
- (ii) By selling washing machines for €650, an electrical store makes a profit of 25%. What did the store pay for these washing machines?

(i) Profit = €576 – €480 = €96

$$\begin{aligned} \text{Percentage profit} &= \frac{\text{Profit}}{\text{Cost Price}} \times \frac{100}{1} \% \\ &= \frac{96}{480} \times \frac{100}{1} = 20\% \end{aligned}$$

The percentage profit = 20%.

(ii) €650 represents 125% of the cost price ... 25% profit margin.

$$\therefore 125\% = €650$$

$$1\% = \frac{€650}{125}$$

$$100\% = \frac{€650}{125} \times \frac{100}{1}$$

$$= €520$$

\therefore the store paid €520 for the washing machines.

Exercise 5.2

1. Express each of the percentages as decimals:

- (i) 6% (ii) 8% (iii) $7\frac{1}{2}\%$ (iv) $9\frac{1}{2}\%$ (v) 10%
 (vi) 13% (vii) $12\frac{1}{2}\%$ (viii) 106% (ix) 112% (x) 84%

2. (i) Increase 200 by 10%.
 (ii) Increase 120 by 20%.
 (iii) Increase 250 by 4%.
 (iv) Decrease 120 by 15%.

3. (i) Find 80% of 400. (ii) Find 5% of 360.
 (iii) Find 185% of 600. (iv) Find $12\frac{1}{2}\%$ of 120.

4. Find the selling price of each of the following:

- (i) An iPad costing €650 + VAT at 23%.
 (ii) A games console costing €120 + VAT at 21%.
 (iii) A set of tyres costing €300 + VAT at $13\frac{1}{2}\%$.
 (iv) A patio furniture set costing €1200 + VAT at 15%.

5. A meal for 4 people cost €110 + VAT at 9%.
 Find the cost of the meal after VAT is added.

6. An hotel bill amounts to €220 after VAT at 10% is added.
 Find the amount of the bill before VAT is added.

7. The selling price of a television set is €984.
 If this includes VAT at 23%, find the price before VAT is added.

8. An electricity bill amounts to €135 after VAT at $12\frac{1}{2}\%$ is added.
 Find the amount of the bill before VAT is added.

9. Write out the amounts that should go in the highlighted boxes marked A to G.

Cost price	Profit or loss	Selling price
€176	€8.80 profit	A
€460	€92 loss	B
€1.80	€0.63 profit	C
D	€5000 loss	€25 000
E	€4.20 profit	€42
F	€2560 loss	€8000
€3680	G	€4800

10. A greengrocer buys oranges for 80c and sells them for €1. Find his percentage profit.
11. Jack bought a painting for €600 and sold it for €660. Find
 (i) his profit (ii) his percentage profit.
12. A menswear store buys jackets for €320 and sells them for €400. Find the percentage profit the store makes.
13. A jeweller bought an engagement ring for €1200 and sold it at a profit of 15%. Find the sale price of the ring.
14. By selling television sets for €720, a store makes a profit of 20%.
 (i) What did the store pay for the television sets?
 (ii) If the store sold the television sets for €690, what percentage profit would it make?
 (iii) If the store sold the television sets for €570, what percentage loss would it make?
15. A shop owner marks everything up by 30%. If she sells an article for €78, find both the cost price and the profit on the article.
16. A garden centre buys azaleas for €11.50 and sells them for €14.95. Calculate the percentage profit it makes.
17. In a sale, the marked prices are reduced by 30%.
 (i) Calculate the sale price of a jacket if the marked price is €350.
 (ii) Find the marked price of a dress if the sale price is €168.
18. By selling a laptop for €1150, a store makes a profit of 25%.
 (i) What did the store pay for the laptop?
 (ii) At what price should the laptop be sold to make a profit of 20%?
19. When an item is sold for €176, the profit is 10% on the cost price. When the selling price is increased to €192, calculate the percentage profit on the cost price.

Answers to OL VAT

Exercise 5.2

1. (i) 0.06 (ii) 0.08 (iii) 0.075 (iv) 0.095
(v) 0.1 (vi) 0.13 (vii) 0.125 (viii) 1.06
(ix) 1.12 (x) 0.84
2. (i) 220 (ii) 144 (iii) 260 (iv) 102
3. (i) 320 (ii) 18 (iii) 1110 (iv) 15
4. (i) €799.50 (ii) €145.20
(iii) €340.50 (iv) €1380
5. €119.90 6. €200
7. €800 8. €120
9. A: €184.80, B: €368, C: €2.43, D: €30 000,
E: €37.80, F: €10 560, G: €1120
10. 25%
11. (i) €60 (ii) 10%
12. 25% 13. €1380
14. (i) €600 each (ii) 15%
(iii) 5%
15. €60; €18 16. 30%
17. (i) €245 (ii) €240
18. (i) €920 (ii) €1104
19. 20% 20. $21\frac{1}{3}\%$
21. (i) €320 (ii) 17%

Applied Arithmetic

chapter

4

Key words

value added tax (VAT)	profit	loss	cost price	selling price
income tax	standard rate	higher rate	standard rate cut-off point	
universal social charge (USC)	PRSI	currency exchange		
compound interest	principal	rate	final amount	depreciation

Section 4.1 VAT – Profit and loss

When dealing with problems involving percentages, it is very important to be able to convert a percentage to a decimal.

When a percentage is expressed as a decimal, the electronic calculator is particularly useful for finding the required percentage.

Some examples are highlighted below:

To find 4%, multiply by 0.04.

To find $6\frac{1}{2}\%$, multiply by 0.065.

To find 12%, multiply by 0.12.

To find 87%, multiply by 0.87.

To find 104%, multiply by 1.04.

To find $106\frac{1}{2}\%$, multiply by 1.065.

To find 112%, multiply by 1.12.

To find 187%, multiply by 1.87.

1. Value-added tax (VAT)

Value-added tax or **VAT** is a government tax which is added to many of the things that we buy. In most shops the marked price of an item includes VAT and so we do not have to calculate it.

Sometimes the prices of more expensive items such as television sets and furniture are given without VAT. In these cases the shopkeeper calculates the VAT amount and adds it to the price of the item.

If an article is priced at €280 plus VAT at 23%, the full price can be worked out as follows:

$$1\% \text{ of } €280 = €2.80$$

$$23\% = €2.80 \times 23 = €64.40$$

$$\therefore \text{the full price} = €280 + €64.40 = €344.40$$



Using a calculator:

$$100\% + 23\% = 123\%$$

To find 123%, we multiply by 1.23

$$\text{Full price} = €280 \times 1.23 = €344.40$$

Example 1

The rate of VAT on electrical goods is 23%.

- (i) Find the selling price of a washing machine priced at €650 + VAT.
- (ii) If the selling price of an *ipad* is €738, find its price before VAT is added on.

(i) $23\% \text{ of } €650 = €650 \times 0.23 = €149.50$
 $\therefore \text{ the selling price of the washing machine} = €650 + €149.50$
OR $\qquad \qquad \qquad = €799.50$

Adding 23% gives 123%.

$123\% \text{ of } €650 = €650 \times 1.23 = €799.50 \dots \text{ as above.}$

- (ii) €738 represents 123% of the price before VAT is added.

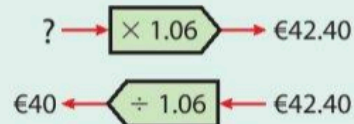
$$\begin{aligned} 123\% &= €738 \\ 1\% &= \frac{738}{123} \\ 100\% &= \frac{738}{123} \times \frac{100}{1} = €600 \end{aligned}$$

You may also divide by 1.23 to get your answer.

Example 2

A rail fare goes up by 6% to €42.40.
What was the old fare?

The old fare is multiplied by 1.06 to get the new fare.
So the new fare has to be divided by 1.06 to get the old fare.
Old fare = $€42.40 \div 1.06 = €40.$



2. Percentage profit and loss

When dealing with percentage profit or loss, we base this percentage on the **cost price** unless otherwise stated.

$$\text{Percentage profit} = \frac{\text{Profit}}{\text{Cost price}} \times \frac{100}{1}; \quad \text{Percentage loss} = \frac{\text{Loss}}{\text{Cost price}} \times \frac{100}{1}$$

- 9.** In a sale, the marked prices are reduced by 30%.
- Calculate the sale price of a jacket if the marked price is €350.
 - Find the marked price of a dress if the sale price is €168.
- 10.** By selling a jacket for €416, a store makes a profit of 30%.
- Find the cost price of the jacket.
 - If the jacket is reduced by 10% in a sale, calculate the percentage profit the store now has on the cost price.
- 11.** By selling a laptop for €1150, a store makes a profit of 25%.
At what price should the laptop be sold to make a profit of 20%?
- 12.** When an item is sold for €176, the profit is 10% on the cost price. When the selling price is increased to €192, calculate the percentage profit on the cost price.
- 13.** A greengrocer buys 30 boxes of strawberries at €5.25 each and sells 28 of them at a profit of 30%. If the remaining two boxes are unsaleable, find his percentage profit on the deal.
- 14.** The price of a games console is €615 which includes VAT at 23%.
- Store A offers a discount of 24% on the selling price.
Store B says it will not charge VAT.
Store C says it will reduce the price before VAT is added by 25% and then charge VAT at 23% on the reduced price.
- In which store is the selling price the cheapest and by how much?
- 15.** Kelly Og's Corn Flakes are sold in standard packs of 500 g for €2.40.
During a promotion, the quantity in a pack is increased by 20%, but the price stays the same.
Calculate the percentage reduction in the price per kilogram during the promotion.



- 16.** A shop advertised "Everything half price in our sale", but also now advertises that there is "An additional 15% off sale prices".
To what percentage reduction on the original price is the new offer equivalent?

Answers to HL VAT

Chapter 4: Applied Arithmetic

Exercise 4.1

1. (i) 0.07 (ii) 0.035 (iii) 0.12 (iv) 0.15
 (v) 0.165 (vi) 1.04 (vii) 1.1 (viii) 1.14
 (ix) 1.25 (x) 0.875
2. (i) 132 (ii) 159 (iii) 570 (iv) 717.5
3. (i) €29.12 (ii) €35
4. €25 5. €639.60 6. €800
7. €180 8. €1600
9. (i) €245 (ii) €240
10. (i) €320 (ii) 17%
11. €1104 12. 20% 13. $21\frac{1}{3}\%$
14. Store C; €6.15 cheaper than Store A
15. $16\frac{2}{3}\%$ 16. Equates to a $57\frac{1}{2}\%$ reduction

OL INCOME TAX

Section 5.5 Income tax

Most wage and salary earners pay a portion of their wages or income to the state.

The money is deducted from their wages by their employers and passed on to the Revenue Commissioners.

Money paid to the state in this way is called **income tax**.

Workers pay income tax on **all** their income at one of two rates.

These rates are called the **Standard Rate** and the **Higher Rate**.

The Standard Rate is generally around 20% and the Higher Rate is around 40%.

In this chapter we will deal only with the standard rate.

At the beginning of the year every worker gets a **tax credit** certificate.

This gives the amount of money that a person can deduct from his **gross tax** every week or month. The employer subtracts the tax credit from the gross tax to get the **tax payable**.

$$\text{Tax payable} = \text{gross tax} - \text{tax credit}$$

A worker earns €800 a week and has a tax credit of €70.

The standard rate of income tax is 20%.

His income tax is calculated as follows:

$$\begin{aligned} \text{Gross tax} &= 20\% \text{ of } \text{€}800 \\ &= \text{€}800 \times 0.2 = \text{€}160 \end{aligned}$$

$$\begin{aligned} \text{Tax payable} &= \text{gross tax} - \text{tax credit} \\ &= \text{€}160 - \text{€}70 \\ &= \text{€}90 \end{aligned}$$

The worker pays €90 in income tax each week.

His **take-home pay** is $\text{€}800 - \text{€}90 = \text{€}710$.

Example 1

A printer has a weekly wage of €860.
 He pays income tax on all his wage at the standard rate of 21%.
 If he has a tax credit of €66 a week, find how much income tax he pays.

$$\begin{aligned} \text{Gross tax} &= 21\% \text{ of } \text{€}860 \\ &= \text{€}860 \times 0.21 = \text{€}180.60 \end{aligned}$$

$$\begin{aligned} \text{Tax payable} &= \text{gross tax} - \text{tax credit} \\ &= \text{€}180.60 - \text{€}66 \\ &= \text{€}114.60 \end{aligned}$$

The printer pays €114.60 income tax each week.

Exercise 5.5

1. Aaron’s weekly wage is €800.
 His tax credit is €90 a week.
 He pays income tax at the rate of 20%.
 Copy and complete the table on the right
 to find Aaron’s take-home pay.

Gross pay	€800
Tax @ 20%
Tax credit	€90
Tax payable
Take-home pay

2. Helen’s gross pay for the year is €34 800.
 Her tax credit is €2585.
 She pays income tax at the rate of 22%.
 Copy and complete the table on the right.

Gross pay	
Tax @ 22%	
Tax credit	
Tax payable	
Take-home pay	

3. Copy and complete the following:
- $$\begin{aligned} \text{Weekly wage} &= \text{€}600 \\ \text{Tax @ 20\% on all income} &= \text{€}600 \times 0.2 = \text{€}... \\ \text{Tax credits} &= \text{€}42 \\ \text{Tax payable} &= \text{€}... - \text{€}42 = \text{€}... \end{aligned}$$
4. Leah has a weekly wage of €770.
 Her weekly tax credit is €72 and the standard rate of tax is 25%.
 How much income tax does she pay each week?
5. Conor has a monthly wage of €3200.
 His monthly tax credit is €280 and the standard rate of tax is 22%.
 Find how much income tax he pays each month.

6. Elaine has a weekly wage of €920.
Her weekly tax credit is €84 and the standard rate of tax is 24%.
How much income tax does she pay each week?
7. Jill has an annual salary of €42 000.
Her annual tax credit is €3600 and the standard rate of tax is 22%.
How much income tax does she pay for the year?
8. A carpenter has a weekly wage of €1050.
His weekly tax credit is €78 and the standard rate of income tax is 24%.
 - (i) Calculate how much income tax he pays each week.
 - (ii) What is his net pay (i.e. take-home pay) for the week?
9. Emma is paid €10.40 per hour for a 35-hour working week.
For each hour over 35 hours, she is paid $1\frac{1}{2}$ times the normal hourly rate.
Her tax credit for the week is €68 and the standard rate of tax is 26%.
If in a particular week she worked 45 hours, find
 - (i) her gross wage for the week
 - (ii) the amount of income tax she pays
 - (iii) her take-home pay for the week.
10. John White is a factory manager and he earns €54 000 a year.
His yearly tax credit is €4300 and the standard rate of income tax is 24%.
He also makes a pension contribution of 6% of his salary.
 - (i) Find his pension contribution for the year.
 - (ii) How much income tax does he pay for the year?
 - (iii) Find his net pay after income tax and pension contribution have been deducted.

OL INCOME TAX Answers

Exercise 5.5

- | | | |
|--------------------|-------------|---------------|
| 1. €730 | 2. €29 729 | |
| 3. €120, €120, €78 | 4. €120.50 | |
| 5. €424 | 6. €136.80 | |
| 7. €5640 | | |
| 8. (i) €174 | (ii) €876 | |
| 9. (i) €520 | (ii) €67.20 | (iii) €452.80 |
| 10. (i) €3240 | (ii) €8660 | (iii) €42 100 |

HL: Income Tax

Section 4.3 Income tax

In this country, wage and salary earners pay income tax on all their incomes at one of two rates.

These rates are called the **Standard Rate** and the **Higher Rate**.

In 2012, the Standard Rate was 20% and the Higher Rate was 41%.

However these rates can change from year to year.

At the beginning of the year, each employed person is given a **tax credit** and a **standard rate cut-off point**. If the standard rate cut-off point is €30 000, this means that the person pays income tax at the standard rate (say 20%) on the first €30 000 of income. Any income above €30 000 is taxed at the higher rate (say 41%). When this income tax has been calculated, it is called the **gross tax**. The person's tax credit is then deducted from the gross tax to give the **tax payable**.

$$\text{Tax payable} = \text{Gross tax} - \text{tax credit}$$

Example 1

A printer has a weekly wage of €800. He pays income tax on all his wages at the standard rate of 20%. If he has a tax credit of €48 a week, find how much income tax he pays.

$$\begin{aligned}\text{Gross tax} &= 20\% \text{ of } €800 \\ &= €800 \times 0.2 = €160\end{aligned}$$

$$\begin{aligned}\text{Tax payable} &= \text{Gross tax} - \text{tax credits} \\ &= €160 - €48 \\ &= €112\end{aligned}$$

∴ he pays €112 income tax each week.

The Higher Rate

The diagram below illustrates the amount of income tax a person with a gross weekly wage of €700 would pay if the standard rate cut-off point is €500 and the tax credit is €80 per week.

Standard Rate (20%)	Higher Rate (40%)
€500 @ 20% = €100	€200 @ 40% = €80

↑
Standard rate cut-off point (€500)

$$\begin{aligned}\text{Gross tax} &= €500 @ 20\% + €200 @ 40\% \\ &= €100 + €80 = €180\end{aligned}$$

$$\begin{aligned}\text{Tax payable} &= \text{Gross tax} - \text{tax credits} \\ &= €180 - €80 = €100\end{aligned}$$

Example 2

A woman's income for the year is €45 000. She has a standard rate cut-off point of €28 000 and a tax credit of €4000. If the standard rate of income tax is 20% and the higher rate is 41%, how much income tax does she pay for the year?

$$\begin{aligned}\text{Gross tax} &= 20\% \text{ of } €28\,000 + 41\% \text{ of the remainder of her salary} \\ &= 20\% \text{ of } €28\,000 + 41\% \text{ of } €17\,000 \quad \dots \text{ Remainder} = €17\,000 \\ &= (28\,000 \times 0.2) + (17\,000 \times 0.41) \\ &= 5600 + 6970 = €12\,570\end{aligned}$$

$$\begin{aligned}\text{Tax payable} &= \text{Gross tax} - \text{tax credit} \\ &= €12\,570 - €4000\end{aligned}$$

∴ Tax payable = €8570

Example 3

A man pays €4500 income tax for the year and he has a tax credit of €2400. If he pays tax at the standard rate of 20% on all his income, calculate his gross income for the year.

$$\text{Tax payable} = \text{Gross tax} - \text{tax credit}$$

$$\Rightarrow \text{€4500} = \text{Gross tax} - \text{€2400}$$

$$\Rightarrow \text{Gross tax} = \text{€4500} + \text{€2400} = \text{€6900}$$

$$\text{Gross tax} = 20\% \text{ of gross income}$$

$$\Rightarrow 20\% \text{ of gross income} = \text{€6900}$$

$$1\% \text{ of gross income} = \text{€} \frac{6900}{20}$$

$$100\% \text{ of gross income} = \text{€} \frac{6900}{20} \times \frac{100}{1}$$

$$\therefore \text{gross income} = \text{€34 500}$$

Universal Social Charge (USC)

The Universal Social Charge (USC) came into effect on 1st January 2011. It replaced other levies which were abolished from that date.

The rates of USC per year and per week are given below:

Income thresholds		
Per year	Rate of USC	Per week
Up to €10 036	2%	Up to €193
From €10 036 to €16 016	4%	From €193 to €308
In excess of €16 016	7%	In excess of €308

PRSI (Pay-Related Social Insurance)

PRSI is another deduction from a person's wage or income.

It is used by the state to pay unemployment benefits and old-age pensions.

The most widely-used rate is 4%.

Example 4

Conor has a weekly wage of €840.

He pays the USC at the usual rates and PRSI at 4%.

He has a tax credit of €65 and pays income tax at the standard rate of 20%.

Calculate his take-home pay after all deductions.

$$\begin{aligned} \text{USC: } & \text{€193 @ 2\%} + (\text{€308} - \text{€193}) @ 4\% + (\text{€840} - \text{€308}) @ 7\% \\ & = \text{€193} \times 0.02 + \text{€115} \times 0.04 + \text{€532} \times 0.07 \\ & = \text{€45.70} \end{aligned}$$

$$\text{PRSI: } \text{€840 @ 4\%} = \text{€840} \times 0.04 = \text{€33.60}$$

$$\begin{aligned} \text{Income tax: Amount} &= \text{Gross tax} - \text{tax credit} \\ &= \text{€840 @ 20\%} - \text{€65} \\ &= \text{€103} \end{aligned}$$

$$\begin{aligned} \text{Total deductions:} &= \text{€45.70} + \text{€33.60} + \text{€103} \\ &= \text{€182.30} \end{aligned}$$

$$\begin{aligned} \text{Take-home pay:} &= \text{€840} - \text{€182.30} \\ &= \text{€657.70} \end{aligned}$$

Exercise 4.3

- Tom's weekly wage is €800.
His tax credit is €90 a week.
He pays income tax at the rate of 20%.
Copy and complete the table on the right to find Tom's take-home pay.

Gross pay	€800
Tax @ 20%
Tax credit	€90
Tax due
Take-home pay

- Helen's gross pay for the year is €34 800.
Her tax credit is €2585.
She pays income tax at the rate of 22%.
Copy and complete the table on the right.

Gross pay	
Tax @ 22%	
Tax credit	
Tax due	
Take-home pay	

- Leah has a weekly wage of €750.
Her weekly tax credit is €72 and the standard rate of tax is 25%.
How much income tax does she pay each week?

4. Conor has a monthly wage of €3200.
His monthly tax credit is €280 and the standard rate of tax is 22%.
Find how much income tax he pays each month.
5. Elaine has a weekly wage of €920.
Her weekly tax credit is €84 and the standard rate of tax is 24%.
How much income tax does she pay each week?
6. Jill has an annual salary of €42 000.
Her annual tax credit is €3600 and the standard rate of tax is 22%.
How much income tax does she pay for the year?
7. A carpenter has a weekly wage of €1050.
His weekly tax credit is €78 and the standard rate of income tax is 24%.
 - (i) Find how much income tax he pays each week.
 - (ii) What is his net pay for the week?
8. Angela has an annual salary of €46 000.
Her standard rate cut-off point is €28 000 and her tax credit is €3200.
If the standard rate of income tax is 20% and the higher rate is 42%, find
 - (i) the gross income tax for the year
 - (ii) the amount of income tax paid for the year.
9. A journalist has a weekly wage of €980.
His standard rate cut-off point is €620 and his tax credits amount to €44. The standard rate of income tax is 20% and the higher rate is 42%.
Find (i) his gross tax for the week
(ii) the amount of tax he pays each week.
10. Niamh has an annual salary of €48 000.
She has a standard rate cut-off point of €34 000 and a tax credit of €4600.
If the standard rate of income tax is 20% and the higher rate is 42%, find how much income tax she pays.
11. The Universal Social Charge (USC) rates are given in the table below:

Per year	Rate of USC	Per week
Up to €10 036	2%	Up to €193
€10 036 to €16 016	4%	€193 to €308
In excess of €16 016	7%	In excess of €308

- (i) Aidan has an annual salary of €57 000.
Calculate his USC for the year.

- (ii) Paula has a weekly wage of €950.
Calculate her USC for the week.
- (iii) A salesperson has an annual salary of €63 000.
Calculate her USC for the year.
- (iv) A plumber has a weekly wage of €1200.
Calculate his USC for the week.
- 12.** A manager has an annual salary of €43 000. Her tax credits amount to €3500 and she pays income tax on all her income at the standard rate of $r\%$.
If she pays €5960 in income tax for the year, find r .
- 13.** A bus driver has a tax credit of €60 a week and pays income tax on all his wages at the standard rate of 20%.
If he pays €140 in income tax for the week, find his gross weekly wage.
- 14.** Linda has a gross annual salary of €48 000 and her standard rate cut-off point is €31 000. The standard rate of income tax is 20% and the higher rate is 35%.
If she pays €7200 in income tax for the year, calculate her tax credit.
- 15.** Helen paid €4400 in income tax for the year. Her tax credits amounted to €2600 and she paid income tax on all her salary at the standard rate of 20%.
Find her gross salary for the year.
- 16.** A man pays €6520 income tax for the year and he has a tax credit of €3600. He pays income tax on all his income at the standard rate of 22%.
Calculate his gross annual salary.
- 17.** Finian has a weekly wage of €900.
His tax credit is €68 and his standard rate cut-off point is €620.
The standard rate of tax is 20% and the higher rate is 42%.
He pays USC at the rates given in Question **11.** above.
He also pays PRSI at the rate of 4% on all his wage.
- How much income tax does he pay for the week?
 - How much does he pay in USC?
 - Calculate how much he pays in PRSI.
 - What is his net pay for the week?
- 18.** John White is a factory manager and earns €54 000 a year.
His yearly tax credit is €4300 and his standard rate cut-off point is €36 000.
The standard rate of tax is 20% and the higher rate is 42%.
He pays USC as in Question **11.** above and PRSI on all his salary at the rate of 4%.
- How much income tax does he pay in the year?
 - How much does he pay in USC?

Net pay is the same as take-home pay.

- (iii) How much in PRSI does he pay?
- (iv) Calculate his take-home pay for the year.
- (v) Express his take-home pay as a percentage of his gross salary.
Give your answer to the nearest whole number.

- 19.** Emma is paid €10.40 per hour for a 35 hour working week. For each hour over 35 hours, she is paid $1\frac{1}{2}$ times the normal hourly rate. Her tax credit for the week is €68 and the standard rate of tax is 26%. She pays USC as in Question 11. above and PRSI at the rate of 5%.

If in a particular week she worked 45 hours, find

- (i) her gross wage for the week
- (ii) the amount of income tax she paid
- (iii) the amount of USC she paid
- (iv) the amount of PRSI she paid
- (v) her net pay for the week.

- 20.** A woman paid €6600 in income tax for the year. She had a tax credit of €4600 and her standard rate cut-off point was €28 000. The standard rate of income tax was 20% and the higher rate was 40%.

- (i) Calculate her gross tax for the year.
- (ii) How much income tax did she pay at the standard rate?
- (iii) How much income tax did she pay at the higher rate?
- (iv) How much income did she earn in excess of €28 000?
- (v) What was the woman's gross income for the year?

ANSWERS

Exercise 4.3

- | | | |
|----------------|--------------|------------|
| 1. €730 | 2. €29 729 | 3. €115.50 |
| 4. €424 | 5. €136.80 | 6. €5640 |
| 7. (i) €174 | (ii) €876 | |
| 8. (i) €13 160 | (ii) €9960 | |
| 9. (i) €275.20 | (ii) €231.20 | |
| 10. €8080 | | |

- | | |
|------------------|-----------------|
| 11. (i) €3308.80 | (ii) €53.40 |
| (iii) €3728.80 | (iv) €70.90 |
| 12. 22 | 13. €1000 |
| 14. €4950 | 15. €35 000 |
| 16. €46 000 | |
| 17. (i) €173.60 | (ii) €49.90 |
| (iii) €36 | (iv) €640.50 |
| 18. (i) €10 460 | (ii) €3098.80 |
| (iii) €2160 | (iv) €38 281.20 |
| (v) 71% | |
| 19. (i) €520 | (ii) €67.20 |
| (iii) €23.30 | (iv) €26 |
| (v) €403.50 | |
| 20. (i) €11 200 | (ii) €5600 |
| (iii) €5600 | (iv) €14 000 |
| (v) €42 000 | |