2nd Year Booklet 2018
Part 2

Topics covered:

1. **Simultaneous Equations**
   a) **Algebraically**
   b) **Graphically**

Ordinary Level

Section 12.1 Simultaneous equations

Consider the equation

$3x + y = 9.$

The values $x = 2$ and $y = 3$ satisfy this equation.
The values $x = 1$ and $y = 6$ also satisfy the equation.
In fact, there are many pairs of values for x and y which satisfy the equation.

Now we take a second equation

$2x - y = 1.$

The values $x = 2$ and $y = 3$ satisfy this equation also.
Thus, the values $x = 2$ and $y = 3$ satisfy both equations

$3x + y = 9$ and $2x - y = 1.$

Only **one** set of values for x and y will satisfy a pair of simultaneous equations.

> When two equations are both satisfied by the same values for x and y, they are said to be simultaneous equations.

Solving a pair of simultaneous equations involves finding the values of x and y that make both equations true.

Example 1

Investigate whether $x = 6$ and $y = 2$ are the correct solutions for these simultaneous equations:

$x + y = 8$ ... □
$x + 2y = 10$ ... □

We substitute 6 for x and 2 for y in each equation:
Equation □: $6 + 2 = 8$ i.e. $8 = 8$; ... correct
Equation □: $6 + 2(2) = 10$ i.e. $10 = 10$; ... correct
Since $x = 6$ and $y = 2$ satisfy both equations, they are the correct solutions.

192

Exercise 12.1

For each of the following pairs of simultaneous equations, solutions are given.
Check if each solution is correct.

1. $x + y = 8$
$x - y = 4$
Solution: $x = 6, y = 2$

2. $2x + y = 11$
$x - 2y = 3$
Solution: $x = 5, y = 1$

3. $3x + 2y = 11$
$2x - 3y = 3$
Solution: $x = 3, y = 1$

4. $4x - y = 5$
$2x + 3y = 8$
Solution: $x = 2, y = 3$

5. $3x - y = 4$
$2x + y = 8$
Solution: $x = 2, y = 4$

6. $3x + y = 14$
$2x - y = 6$
Solution: $x = 4, y = 2$

7. $x + 2y = 4$
$x - y = 5$
Solution: $x = 2, y = 1$

8. $2x + 3y = 3$
$x - 4y = 7$
Solution: $x = 3, y = -1$

9. $2x - y = -8$
$x + 2y = 10$
Solution: $x = -2, y = 4$

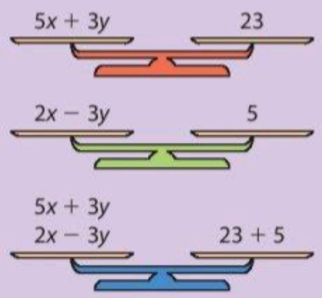
Section 12.2 Solving simultaneous equations

There are many ways of solving simultaneous equations.
One method is called the **elimination method**.
In this method, we 'eliminate' one of the variables.
This method is discussed in the box below.

Discussion

$5x + 3y = 23$
$2x - 3y = 5$ are simultaneous equations.

We can illustrate them using scales.



If we add the left-hand sides and then add the right-hand sides, would the scales still balance? **Discuss.**

What is the left-hand side total?
What is the right-hand side total?
How can you use this to find x? **Discuss.**
What is the value of x?
How can you use this to find y? **Discuss.**

From the 'discussion' above, it can be seen that it is easy to solve simultaneous equations when one of the variables can be eliminated by adding or subtracting the two equations.

If a variable cannot be eliminated by simply adding or subtracting, then we have to multiply one or more of the equations by a number to make the elimination possible.

Example 1

Solve the simultaneous equations $\begin{array}{l} x + 2y = 10 \\ 2x - y = 5 \end{array}$

(To solve these equations, we must make the number of *x*'s or the number of *y*'s equal. We then add or subtract as necessary.
For convenience, we will call the first equation ① and the second equation ②.)

Equation ①: $x + 2y = 10$
Equation ② × 2: $4x - 2y = 10$
Adding: $5x = 20$
$\therefore \quad x = 4$

Substituting $x = 4$ in ① we get: $4 + 2y = 10$
$2y = 6$
$\therefore \quad y = 3$

The solution is $x = 4$ and $y = 3$.

(Note: It is important to check that $x = 4$ and $y = 3$ satisfy both equations.)

Example 2

Solve these simultaneous equations $\begin{array}{l} 2x - 5y = 9 \\ 3x + 2y = 4 \end{array}$

We number the equations ① and ② for convenience.
$2x - 5y = 9$ ①
$3x + 2y = 4$ ②

We now multiply equation ① by 3 and equation ② by 2 to equate the number of *x*'s.

① × 3: $6x - 15y = 27$
② × 2: $6x + 4y = 8$
Subtract: $-19y = 19$
$19y = -19$ ... multiply both sides by -1
$y = -1$

We now substitute -1 for *y* in equation ①

$2x - 5y = 9$
$y = -1 \Rightarrow 2x + 5 = 9$
$\Rightarrow 2x = 4 \Rightarrow x = 2$

$\therefore \quad x = 2$ and $y = -1$ are the solutions.

Exercise 12.2

Solve the following simultaneous equations:

1. $x + y = 9$
   $x - y = 3$
2. $2x + y = 8$
   $3x - y = 2$
3. $2x - y = 4$
   $x + y = 5$
4. $3x + y = 7$
   $x + y = 5$
5. $4x + y = 17$
   $2x + y = 11$
6. $x + 2y = 4$
   $x - y = 1$
7. $x + 2y = 7$
   $2x + y = 8$
8. $2x - 3y = 1$
   $2x + 5y = 9$
9. $2x + 3y = 3$
   $x - 4y = 7$
10. $2x - y = -8$
    $x + 2y = 6$
11. $4x - y = 10$
    $x - y = 1$
12. $4x - y = -9$
    $2x - 3y = -7$
13. $3x - y = 3$
    $x + 3y = 11$
14. $2x + y = -2$
    $x + 3y = 9$
15. $2x - 3y = 14$
    $2x - y = 10$
16. $3x + 4y = 5$
    $2x - 3y = 9$
17. $3x + y = 5$
    $5x - 4y = -3$
18. $2x - y = 12$
    $3x + 2y = 11$
19. $3x + 4y = 10$
    $4x + y = 9$
20. $3x - 2y = 13$
    $4x + 3y = 6$
21. $3x + 5y = 6$
    $2x + 3y = 5$
22. $4x + 3y = 19$
    $3x - 2y = -7$
23. $2x - 5y = 1$
    $5x + 3y = 18$
24. $3x = 5y + 13$
    $2x + 5y = -8$
25. $x + 2y = 13$
    $3x = 5y + 6$
26. $7x + 2 = 2y$
    $3x = 14 - y$
27. $2x - 5y = 3$
    $x = 3y + 1$
28. $3x = 22 + 2y$
    $5y = 2x$
29. $2x - 3y = 8$
    $3x + 4y = -22$
30. $2x - 5y = 22$
    $3x + 7y = 4$

Chapter 12: Simultaneous Equations

Exercise 12.1

1. Correct **2.** Correct **3.** Correct
4. Incorrect **5.** Incorrect **6.** Correct
7. Incorrect **8.** Correct **9.** Incorrect

Exercise 12.2

1. $x = 6, y = 3$ **2.** $x = 2, y = 4$ **3.** $x = 3, y = 2$
4. 1, 4 **5.** 3, 5 **6.** 2, 1
7. 3, 2 **8.** 2, 1 **9.** 3, −1
10. −2, 4 **11.** 3, 2 **12.** −2, 1
13. 2, 3 **14.** −3, 4 **15.** 4, −2
16. 3, −1 **17.** 1, 2 **18.** 5, −2
19. 2, 1 **20.** 3, −2 **21.** 7, −3
22. 1, 5 **23.** 3, 1 **24.** 1, −2
25. 7, 3 **26.** 2, 8 **27.** 4, 1
28. 10, 4 **29.** −2, −4 **30.** 6, −2

Section 14.1 Solving simultaneous equations

Consider the equation

$3x + y = 9.$

The values $x = 2$ and $y = 3$ satisfy this equation.
The values $x = 1$ and $y = 6$ also satisfy the equation.
In fact, there are many pairs of values for x and y which satisfy the equation.

Now we take a second equation

$2x - y = 1.$

The values $x = 2$ and $y = 3$ satisfy this equation also.
Thus, the values $x = 2$ and $y = 3$ satisfy both equations

$3x + y = 9$ and $2x - y = 1.$

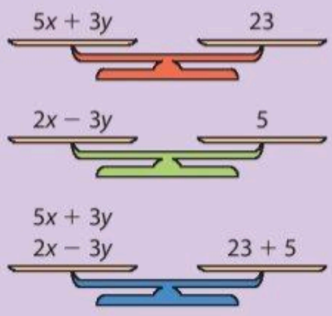
> When two equations are both satisfied by the same values for x and y, they are said to be simultaneous equations.

Only **one** set of values for x and y will satisfy a pair of simultaneous equations.
There are many ways of solving simultaneous equations.
One method is called the **elimination method**.
In this method, we 'eliminate' one of the variables.
This method is discussed in the box below.

Discussion

$5x + 3y = 23$
$2x - 3y = 5$ are simultaneous equations.

We can illustrate them using scales.



If we add the left-hand sides and add the right-hand sides, would the scales still balance? **Discuss.**

What is the left-hand side total?
What is the right-hand side total?
How can you use this to find x? **Discuss.**
What is the value of x?
How can you use this to find y? **Discuss.**

From the 'discussion' on the previous page, it can be seen that it is easy to solve simultaneous equations if one of the variables can be eliminated by adding or subtracting the two equations.

If a variable cannot be eliminated by adding or subtracting, then we have to multiply one or more of the equations by a number to make the elimination possible.

Example 1

Solve the simultaneous equations $x + 2y = 10$, $2x - y = 5$

(To solve these equations, we must make the number of x's or the number of y's equal. We then add or subtract as necessary.
For convenience, we call the first equation ① and the second equation ②.)

Equation ①: $x + 2y = 10$
Equation ② × 2: $4x - 2y = 10$
Adding: $5x = 20$
$\therefore x = 4$

Substituting $x = 4$ in ① we get: $4 + 2y = 10$
$2y = 6$
$\therefore y = 3$

The solution is $x = 4$ and $y = 3$.

(Note: It is important to check that $x = 4$ and $y = 3$ satisfy both equations.)

Example 2

Solve the simultaneous equations $2x - 5y = 9$, $3x + 2y = 4$

We number the equations ① and ② for convenience.
$2x - 5y = 9$ ①
$3x + 2y = 4$ ②

We now multiply equation ① by 3 and equation ② by 2 to equate the number of x's.

① × 3: $6x - 15y = 27$
② × 2: $6x + 4y = 8$
Subtract: $-19y = 19$
$19y = -19$
$y = -1$

We now substitute -1 for y in equation ①

$2x - 5y = 9$
$y = -1 \Rightarrow 2x + 5 = 9$
$\Rightarrow 2x = 4 \Rightarrow x = 2$

$\therefore x = 2$ and $y = -1$ are the solutions.

Equations containing fractions

Here are two simultaneous equations:

$$3x - 2y = 19 \ldots ①$$

$$\frac{x}{3} + \frac{y}{2} = 5 \quad \ldots ②$$

To solve these equations, we 'get rid of the fractions' in equation ② by multiplying each term by 6 (the LCM of 3 and 2).

Thus, equation ② becomes: $6\left(\frac{x}{3}\right) + 6\left(\frac{y}{2}\right) = 6 \times 5$

i.e. $2x + 3y = 30$

The simultaneous equations can now be solved as in Example 2, on the previous page.

Exercise 14.1

1. $2x + y = 8$
$3x - y = 2$

2. $3x + y = 14$
$2x - y = 6$

3. $3x + y = 7$
$x + y = 5$

4. $4x + y = 17$
$2x + y = 11$

5. $8x - 2y = 10$
$5x - 2y = 4$

6. $3x - 2y = 8$
$x + y = 6$

7. $3x + y = 13$
$x - 2y = -5$

8. $x + 2y = 12$
$3x - 5y = 3$

9. $3x - y = 3$
$x + 3y = 11$

10. $2x - 3y = 14$
$2x - y = 10$

11. $3x + y = 5$
$5x - 4y = -3$

12. $2x - y = 12$
$3x + 2y = 11$

13. $x + 2y = 12$
$3x - 5y = 3$

14. $x - 3y = 1$
$4x + y = 30$

15. $3x + 7y = 20$
$x - 2y = -2$

16. $3x - 2y = 17$
$4x + 3y = 0$

17. $4x + 3y = 22$
$5x - 4y = 43$

18. $5x + 2y = 5$
$6x - y = 23$

19. $3x + 4y = 23$
$y = 2x + 3$

20. $5y = 16 - 4x$
$6x = 13 - 2y$

21. $5x + y = 19$
$2x - y = 2y - 2x$

22. $2x - 3y = 24$
$\frac{5x}{3} - \frac{y}{2} = 12$

23. $2x - y = 18$
$\frac{x}{3} - \frac{y}{4} = 2$

24. $x + y = 5$
$\frac{4x}{3} - \frac{y}{2} = -8$

25. $3x - 2y = 19$
$\frac{x}{3} + \frac{y}{2} = 5$

26. $9 = 2x - 3y$
$3y + x = 9$

27. $3x = y - 4$
$3y = 34 - 2x$

28. $\frac{x}{2} + \frac{y}{5} = 4$
$\frac{x}{4} + \frac{y}{2} = 6$

29. $4x + y = 17$
$\frac{x - 3}{4} + \frac{y}{2} = \frac{5}{2}$

30. $3x + y = 19\frac{1}{2}$
$x - 2y = 3$

Section 14.2 Solving simultaneous equations graphically

Discussion

Here are two lines:

$$x + y = 1 \quad \ldots ①$$
$$2x - y = -4 \ldots ②$$

We can draw each line by taking two points on the line.

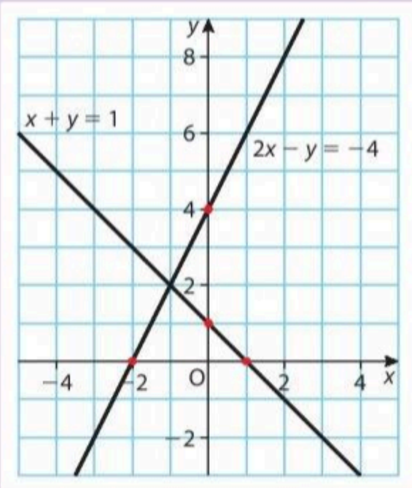
For line ①: $x = 0 \Rightarrow y = 1$
$y = 0 \Rightarrow x = 1$

$\therefore$ (0, 1) and (1, 0) are two points on the line.

For line ②: $x = 0 \Rightarrow -y = -4 \Rightarrow y = 4$
$y = 0 \Rightarrow 2x = -4 \Rightarrow x = -2$

$\therefore$ (0, 4) and (−2, 0) are two points on the line.

The two lines are drawn on the right.



What are the coordinates of the point which lies on *both* lines?

Solve the simultaneous equations $\begin{array}{l} x + y = 1 \\ 2x - y = -4 \end{array}$ using the adding or subtracting equations method.

Compare the values of *x* and *y* you found with the coordinates of the point where these lines meet.

What do you notice? **Discuss.**

From the 'discussion' above, you will have discovered that two simultaneous equations may be solved by drawing graphs of the two equations (lines) and then reading the *x*-value and *y*-value of their point of intersection.

Using simultaneous equations to solve problems

Simultaneous equations are particularly useful for solving problems which have two unknowns. Generally, two different pieces of information enable us to write down two equations.

The following examples illustrate this procedure.

Example 1

The sum of two numbers is 19.
When twice the second number is taken from three times the first number, the result is 22.
Find the two numbers.

Let x and y be the numbers.

Equation ①: $x + y = 19$
Equation ②: $3x - 2y = 22$

Equation ① × 2: $2x + 2y = 38$
Equation ②: $3x - 2y = 22$
Add: $5x = 60$
$x = 12$

Substituting 12 for x in equation ①, we get: $12 + y = 19$
$y = 19 - 12$
$y = 7$

The two numbers are 12 and 7.

Example 2

Tickets to a movie cost €8 or €10.
If 300 tickets were sold and the total amount of money collected was €2640, how many of each type of ticket were sold?

Let x = number of €8 tickets sold, and
y = number of €10 tickets sold.

① Number: $x + y = 300$
② Money: $8x + 10y = 2640$

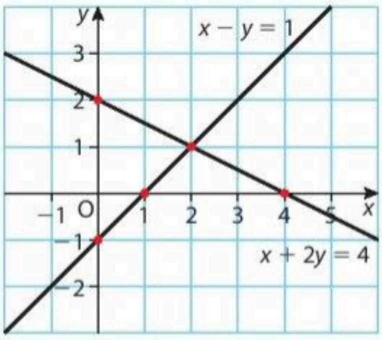
① × 8: $8x + 8y = 2400$
②: $8x + 10y = 2640$
Subtract: $-2y = -240$
$\Rightarrow 2y = 240$
$\Rightarrow y = 120$

Substituting 120 for y in equation ①, we get:
$x + y = 300$
$y = 120$: $x + 120 = 300$
$x = 180$
$\therefore$ $x = 180$ and $y = 120$

$\therefore$ 180 €8 tickets and 120 €10 tickets were sold.

Exercise 14.2

1. The given diagram shows the graphs of the lines $x - y = 1$ and $x + 2y = 4$.



(i) Write down the point of intersection of the two lines.
(ii) Solve the simultaneous equations $x - y = 1$ and $x + 2y = 4$.
(iii) Explain the connection between your answers in (ii) to the answer you found in (i) above.

2. Graph the lines $2x - y = 1$ and $x - y = -1$.
Hence write down the point of intersection of the two lines.

3. Solve these simultaneous equations by drawing graphs:

(i) $2x - y = -1$
$x - y = 1$

(ii) $2x - y = 1$
$x - y = -2$

4. Use graphs to solve each pair of simultaneous equations.

(i) $x + y = 6$
$x - y = 2$
Draw axes for x and y from -3 to 7.

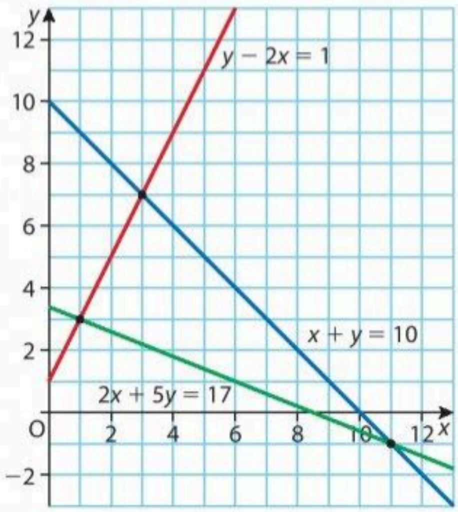
(ii) $y + 2x = 5$
$x - y = 1$
Draw axes for x and y from -2 to 6.

Check each solution by substituting into the equations.

5. Use the given graphs to solve each pair of simultaneous equations.

(i) $x + y = 10$
$y - 2x = 1$
(ii) $2x + 5y = 17$
$y - 2x = 1$
(iii) $x + y = 10$
$2x + 5y = 17$

Check each solution by substituting into the equations.



6. (i) Use the graphs to solve each pair of simultaneous equations.

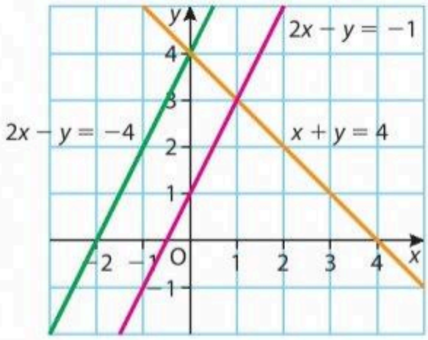
(a) $2x - y = -4$
$x + y = 4$

(b) $2x - y = -1$
$x + y = 4$

Check each solution by substituting into the equations.

(ii) How do you know there is no solution to the following pair of simultaneous equations?

$2x - y = -4$
$2x - y = -1$



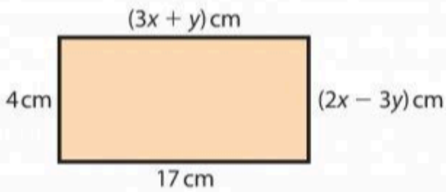
7. The sum of two numbers is 9. If twice the first number is added to three times the second, the answer is 15. Find the two numbers.

8. The difference of two numbers is 7. When three times the smaller number is taken from twice the larger, the result is 11. Find the two numbers.

9. Find two numbers such that the first number added to three times the second is 31, while three times the first less twice the second is 16.

10. Examine the rectangle shown and write down two equations in x and y.

Now solve these equations to find the value of x and the value of y.



11. Three nuts and six bolts have a combined weight of 72 g.
Four nuts and five bolts have a combined weight of 66 g.
Find the combined weight of one nut and one bolt.

12. Three chocolate bars and four chocolate eggs weigh 465 grams.
Three chocolate bars and two chocolate eggs weigh 315 grams.

(i) Which of these pairs of equations is correct for the chocolate bars and eggs?

A
$3b + 2e = 465$
$3b + 4e = 315$

B
$b + 4e = 465$
$b + 2e = 315$

C
$3b + 4e = 465$
$3b + 2e = 315$

(ii) Solve the pair of simultaneous equations that is correct to find the values of b and e.

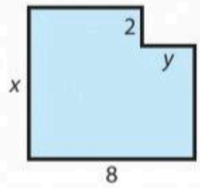
13. A bag contains 34 coins, all of them either 5c or 10c coins. If the value of the money in the bag is €2.40, find how many of each coin the bag contains.
[Hint: Convert €2.40 to cents.]

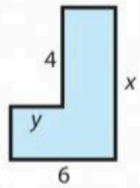
14. Two small mugs and one large mug weigh 758 grams.
Four small mugs and three large mugs weigh 1882 grams.

The weight of a small mug is *s* grams and the weight of a large mug is *l* grams.

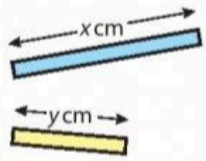
Write down two equations connecting *s* and *l*.
Solve these simultaneous equations to find the weight of each size of mug.

15. In the given two figures, all the angles are right angles and the distances are in centimetres. If the area of the figure on the left is 45 cm^2 and the area of the second figure is 25 cm^2, find the values of *x* and *y*.





16. The length of a blue rod is *x* cm.



The length of a yellow rod is *y* cm.

(i) The total length of 2 blue rods and 3 yellow rods is 33 cm.

Write an equation for this diagram.

(ii) The total length of 4 blue rods and 2 yellow rods is 46 cm.

Write an equation for this diagram.

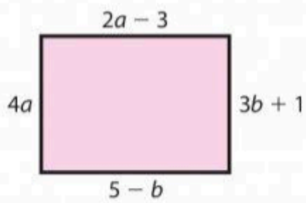
(iii) Solve your equations simultaneously to find the values of *x* and *y*.

(iv) What is the total length of 1 blue and 6 yellow rods?

17. The diagram shows a rectangle.
All sides are measured in centimetres.

(i) Write down a pair of simultaneous equations in *a* and *b*.

(ii) Solve your pair of simultaneous equations to find *a* and *b*.



18. A car-hire company charges €*x* per day to hire a car and then adds a charge of €*y* for each kilometre travelled.

John hires a car for 2 days and travels 250 km. He is charged €200.

Emer hires a car for 5 days and travels 620 km. She is charged €498.

Write two equations in *x* and *y* and solve them to find the charge per day and the charge for each kilometre.

Chapter 14: Simultaneous Equations

Exercise 14.1

1. $x = 2, y = 4$
2. $x = 4, y = 2$
3. 1, 4
4. 3, 5
5. 2, 3
6. 4, 2
7. 3, 4
8. 6, 3
9. 2, 3
10. 4, −2
11. 1, 2
12. 5, −2
13. 6, 3
14. 7, 2
15. 2, 2
16. 3, −4
17. 7, −2
18. 3, −5
19. 1, 5
20. $1\frac{1}{2}$, 2
21. 3, 4
22. 6, −4
23. 15, 12
24. −3, 8
25. 9, 4
26. 6, 1
27. 2, 10
28. 4, 10
29. 3, 5
30. 6, $1\frac{1}{2}$

Exercise 14.2

1. (i) (2, 1) (ii) $x = 2, y = 1$
   (iii) Same
2. (2, 3)
3. (i) −2, −3 (ii) 3, 5
4. (i) 4, 2 (ii) 2, 1
5. (i) 3, 7 (ii) 1, 3 (iii) 11, −1
6. (i) (a) 0, 4 (b) 1, 3
   (ii) Parallel lines, therefore no point of intersection
7. 12, −3
8. 10, 3
9. 10, 7
10. $x = 5, y = 2$
11. 14 g
12. (i) Ⓒ (ii) $b = 55$ g, $e = 75$ g
13. Twenty 5c coins and fourteen 10c coins
14. Small mug – 196 g, large mug – 366 g
15. $x = 6\frac{1}{2}$ cm, $y = 3\frac{1}{2}$ cm
16. (i) $2x + 3y = 33$ (ii) $4x + 2y = 46$
    (iii) $x = 9, y = 5$ (iv) 39 cm
17. (i) $2a + b = 8$, $4a - 3b = 1$
    (ii) $a = 2\frac{1}{2}$, $b = 3$
18. €50, 40 c
19. 40, 12
20. 42, 36
21. □ = 4, ♡ = 20, ◇ = 2
22. $2x - y = 3$, $x + 2y = -6$