Question 1 (25 marks)

A survey of 50 Leaving Certificate candidates in 2014, randomly selected in the Dublin region, found that they had a mean mark of 374 in a certain subject. The standard deviation of this sample was 45.

(a) Find the 95% confidence interval for the mean mark in the subject, in the Dublin region. Interpret this interval.



(b) The mean mark in the subject for all Leaving Certificate candidates, in 2014, was 385 and the standard deviation was 45. John suggests that the mean mark in the Dublin region is not the same as in the whole country. Test this hypothesis using a 5% level of significance. Clearly state your null hypothesis, your alternative hypothesis and your conclusion.



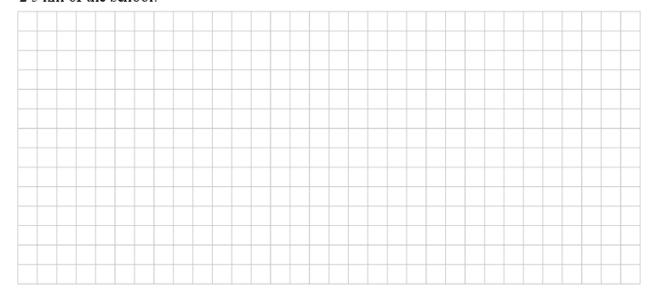
Question 2 (50 marks)

The principal of a large school claims that the average distance from a student's home to the school is 3.5 km. In order to test this claim, a sample of 60 students from the school was randomly selected. The students were asked how far from the school they lived. The mean distance from these students' homes to the school is 3.7 km with a standard deviation of 0.5 km.

(a) Test the principal's claim using a 5% level of significance. Clearly state your null hypothesis, your alternative hypothesis and your conclusion.



(b) In the above sample of 60 students, 20% of them lived within 2.5 km of the school. Find the 95% confidence interval for the proportion of students from that school who live within 2.5 km of the school.



(c) Data from 10 years ago shows that, at that time, 26% of the student population lived within 2·5 km of the school. Based on your answer to part (b) is it possible to conclude, at the 5% level of significance, that the proportion of students living within 2·5 km of the school has changed since that time? Explain your answer.



(d) A statistician wishes to estimate, with 95% confidence, the proportion of students who live within a certain distance of the school. She wishes to be accurate to within 10 percentage points of the true proportion. What is the minimum sample size necessary for the statistician to carry out this analysis?



Question 3 (25 marks)

(a) The mean lifetime of light bulbs produced by a company has, in the past, been 1500 hours. A sample of 100 bulbs, recently produced by the company, had a mean lifetime of 1475 hours with a standard deviation of 110 hours. Test the hypothesis that the mean lifetime of the bulbs has not changed, using a 0.05 level of significance.



(b) Find the *p*-value of the test you performed in part (a) above and explain what this value represents in the context of the question.



Question 4 (50 marks)

A car rental company has been using *Evertread* tyres on their fleet of economy cars. All cars in this fleet are identical. The company manages the tyres on each car in such a way that the four tyres all wear out at the same time. The company keeps a record of the lifespan of each set of tyres. The records show that the lifespan of these sets of tyres is normally distributed with mean 45 000 km and standard deviation 8000 km.

(a) A car from the economy fleet is chosen at random. Find the probability that the tyres on this car will last for at least 40 000 km.



(b) Twenty cars from the economy fleet are chosen at random. Find the probability that the tyres on at least eighteen of these cars will last for more than 40 000 km.



(c) The company is considering switching brands from *Evertread* tyres to *SafeRun* tyres, because they are cheaper. The distributors of *SafeRun* tyres claim that these tyres have the same mean lifespan as *Evertread* tyres. The car rental company wants to check this claim before they switch brands. They have enough data on *Evertread* tyres to regard these as a known population. They want to test a sample of *SafeRun* tyres against it.

The company selects 25 cars at random from the economy fleet and fits them with the new tyres. For these cars, it is found that the mean life span of the tyres is 43 850 km.

Test, at the 5% level of significance, the hypothesis that the mean lifespan of *SafeRun* tyres is the same as the mean of *Evertread* tyres. State clearly what the company can conclude about the tyres.

