W M S 2 3

examCRAFT
...supporting education

PRE-LEAVING CERTIFICATE EXAMINATION, 2008

APPLIED MATHEMATICS

MARKING SCHEME

HIGHER AND ORDINARY LEVEL

35 Finglas Business Park, Tolka Valley Road, Finglas, Dublin 11
T: 01 808 1494, F: 01 836 2739, E: info@examcraft.ie, W: www.examcraft.ie

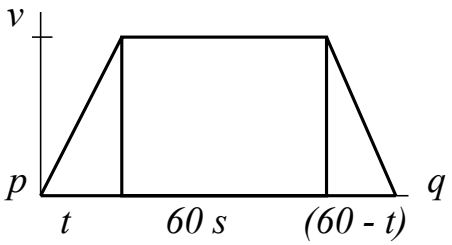
Higher Solutions

1. **(a)**

$$a = \frac{v}{t} \Rightarrow V = 3t$$

$$d = \frac{v}{60 - t} \Rightarrow V = 2(60 - t)$$

$$\Rightarrow 3t = 120 - 2t \Rightarrow 5t = 120 \Rightarrow t = 24$$



(i) $Dist = \frac{1}{2}(24)(72) + (72)(60) + \frac{1}{2}(36)(72) = 6{,}480 \text{ m}$ **10 A (3)**

(ii) $t(t + 48) = 6{,}480$

$$\Rightarrow t^2 + 48t - 6480 = 0$$

$$(t + 108)(t - 60) = 0$$

$$\Rightarrow t = 60s$$ **10 A (3)**

(b) $Sp = u(t + 3) - \frac{1}{2}g(t + 3)^2$ **10 A (3)**

$$Sq = 1.5ut - \frac{1}{2}gt^2$$

$Sp = Sq$ **10 A (3)**

$$u(t + 3) - \frac{1}{2}g(t + 3)^2 = 1\frac{1}{2}ut - \frac{1}{2}gt^2$$

$$\Rightarrow ut + 3u - \frac{1}{2}gt^2 - 3gt - \frac{9}{2}g = 1\frac{1}{2}ut - \frac{1}{2}gt^2$$

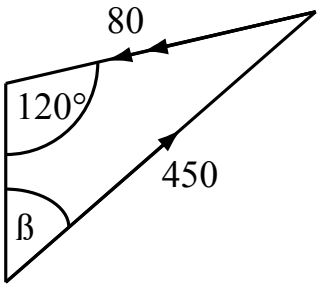
$$\Rightarrow \frac{3}{2}ut - ut - 3u + \frac{9}{2}g = -3gt$$

$$\frac{1}{2}ut + 3gt = 3u - \frac{9}{2}g$$

$$\Rightarrow t = \frac{6u - 9g}{u + 6g}$$

10 A (3)

2.



(i)

$$Outward: \frac{\sin B}{80} = \frac{\sin 120°}{450}$$

$$\Rightarrow \hat{B} = 8.86°$$

$$\Rightarrow N8.86°E$$

10 A (3)

$$\text{Re}turn: \frac{\sin B}{80} = \frac{\sin 60°}{450°} \Rightarrow \hat{B} = 8.86°$$

$$\Rightarrow S.8.86°E$$

10 A (3)

(ii)

$$Outward: |V_p| = \sqrt{80^2 + 450^2 - 2\times 80\times 450\times \cos 51.14°} = 404.63$$

$$\text{Re}turn: |V_p| = \sqrt{80^2 + 450^2 - 2\times 80\times 450\times \cos 111.14°} = 484.63$$

$$\Rightarrow Time: 2500\left(\frac{1}{404.63} + \frac{1}{484.63}\right) = 11.3 \text{ hrs}$$

20 A (3)

(iii)

$$2\times \frac{2500}{450} = 11\frac{1}{9} \text{ hrs}$$

10 A (3)

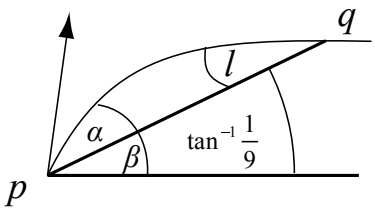
3. **(i)** $\vec{r} = \left(u\cos\alpha t - \frac{1}{2}g\sin\beta t^2\right)\vec{i} + \left(u\sin\alpha t - \frac{1}{2}g\cos\beta t^2\right)\vec{j}$

$\vec{v} = (u\cos\alpha - gt\sin\beta)\vec{i} + (u\sin\alpha - gt\cos\beta)\vec{j}$

$Sy = 0 = u\sin\alpha t - \frac{1}{2}g\cos\beta t^2 = 0 \Rightarrow t = \frac{2u\sin\alpha}{g\cos\beta}$

10 A (3)

$\tan l = \frac{-v_y}{v_x} (= \tan\beta)$



$v_y = u\sin\alpha - g\cos\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right) = -u\sin\alpha$

$v_x = u\cos\alpha - g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right) = u\cos\alpha - 2u\sin\alpha\tan\beta$

$\therefore \frac{u\sin\alpha}{u\cos\alpha - 2u\sin\alpha\left(\frac{1}{9}\right)} = \frac{9u\sin\alpha}{9u\cos\alpha - 2u\sin\alpha} = \frac{1}{9}$

10 A (3)

$81u\sin\alpha = 9u\cos\alpha - 2u\sin\alpha$

$\Rightarrow 91\sin\alpha = 9\cos\alpha \Rightarrow \tan\alpha = \frac{9}{91}$

10 A (3)

(ii) $v\vec{i} = u\cos\sigma - 2u\sin\sigma\tan\beta = 0$ (when striking at right angles) **10 A (3)**

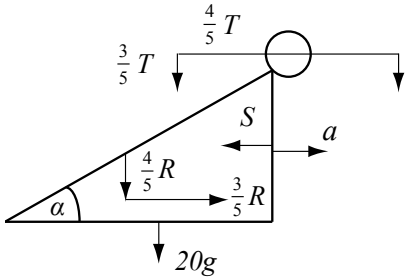
$\cos\sigma - 2\sin\sigma\left(\frac{1}{9}\right) = 0$

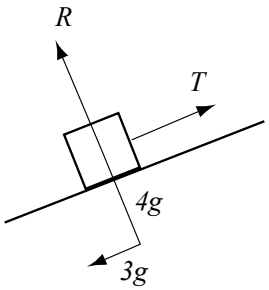
$\Rightarrow \tan\sigma = \frac{9}{2}$

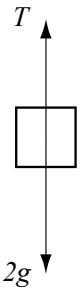
$\Rightarrow \sigma = \tan^{-1}\left(\frac{9}{2}\right)$

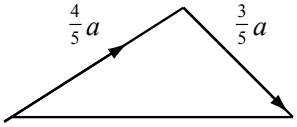
10 A (3)

4. Force diagrams









Wedge: 10 A (3)
5 kg: 10 A (3)
2 kg: 10 A (3)

5 kg Mass

$(\leftrightarrow)$ Parr to plane

$$3g - T = 5\left(f - \frac{4}{5}a\right)$$

$$3g - T = 5f - 4a \qquad (A)$$

$(\updownarrow)$ Perp to plane

$$4g - R = 5\left(\frac{3}{5}a\right)$$

$$4g - R = 3a \ (B)$$

2 kg Mass

$(\updownarrow)$ $T - 2g = 2f \ (C)$

$(\leftrightarrow)$ $S = 2a \ (D)$

Wedge:

$(\leftrightarrow)$ $\frac{3}{5}R - \frac{4}{5}T - S = 20a \ (E)$ **10 A (3)**

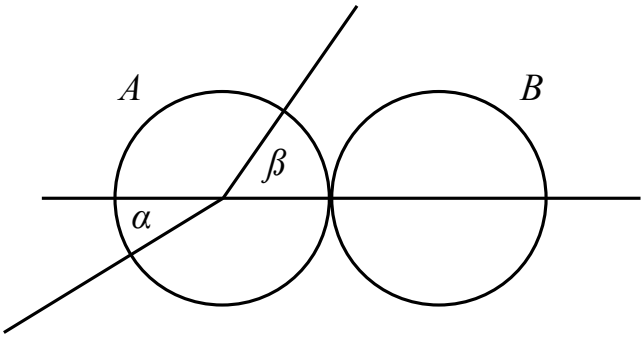
Eliminate T *from* (A) and $(C) \Rightarrow f = \frac{4a+g}{7} \ (F)$

Subst B, C, D and F into (E)

$$\frac{3}{5}(4g - 3a) - \frac{4}{5}\left(2g + \frac{2(4a+g)}{7}\right) - 2a = 20a$$

$\Rightarrow a = \frac{4}{173}g$ **10 A (3)**

5.



	Mass	Before	After
A:	m	$u\cos\alpha\vec{i} + u\sin\alpha\vec{j}$	$w\cos\beta\vec{i} + w\sin\beta\vec{j}$
B:	m	$o\vec{i} + o\vec{j}$	$p\vec{i} + o\vec{j}$

COM: $m(u\cos\alpha + 0) = m(w\cos\beta + p) \Rightarrow w\cos\beta + p = u\cos\alpha$ **5 A (3)**

NEL: $\dfrac{p - w\cos\beta}{-u\cos\alpha} = -e \Rightarrow \begin{matrix} w\cos\beta - p = -eu\cos\alpha \\ 2w\cos\beta = u\cos\alpha(1-e) \end{matrix}$ **5 A (3)**

$\alpha = \cos^{-1}\dfrac{5}{\sqrt{42}},\ \beta = \cos^{-1}\dfrac{3}{7},\ e = \dfrac{3}{5}$

$\dfrac{6w}{7} = \dfrac{5u}{\sqrt{42}} \times \dfrac{2}{5} \Rightarrow w = \dfrac{\sqrt{42}u}{18},\ p = \dfrac{2\sqrt{42}u}{21}$ **10 A (3)** }

(i) **10 A (3)**

(ii) $K.E_{Before} = \dfrac{1}{2}mu^2$ **5 A (2)**

$K.E._{After} = \dfrac{1}{2}mw^2 + \dfrac{1}{2}mp^2 = \dfrac{1}{2}(w^2 + p^2)m$

$= \dfrac{1}{2}m\left(\dfrac{42u^2}{324} + \dfrac{168u^2}{441}\right) = \dfrac{1}{2}m\left(\dfrac{193}{378}\right)u^2$ **5 A (2)**

$Loss\ in\ K.E. = \dfrac{185}{756}u^2$

$\%\ Loss\ in\ K.E.\ = 24.47\%$ **10 A (3)**

6. **(a)**

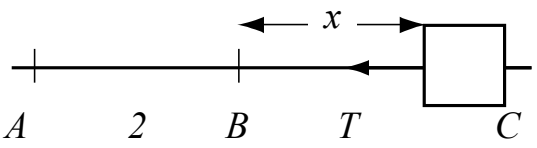
$$T = \frac{2\pi}{w} = \frac{\pi}{2} \Rightarrow w = 4$$

$$V_{\max} = wA = 4A = 10 \Rightarrow A = 2.5$$ **10 A (3)**

$$v^2 = w^2\left(A^2 - x^2\right) = 4^2\left(2.5^2 - 2^2\right) = 36$$

$$\therefore v = 6$$ **10 A (3)**

(b)



(i) **Force in direction of x increasing** $= -T$

$$3\ddot{x} = -15x$$

$$\ddot{x} = -5x$$

$\therefore S.H.M.$ **about** $x = 0$ with $w = \sqrt{5}$ **10 A (3)**

$$P.T = \frac{2\pi}{w} = \frac{2\pi}{\sqrt{5}}$$

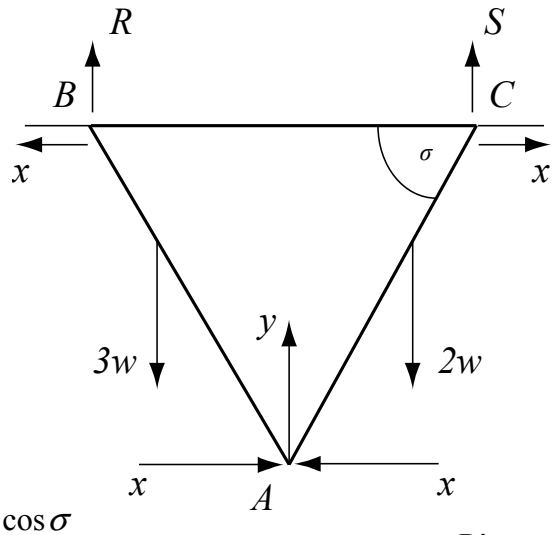
(ii) ***Time*** $= \frac{T}{4} = \frac{2\pi}{4\sqrt{5}} = \frac{\pi}{2\sqrt{5}}$ **10 A (3)**

(iii) $v = wa = \sqrt{5} \times \left(\frac{1}{2}\right)$

It travels 4 m with uniform speed

$$\text{Time} = \frac{4}{\sqrt{5}\left(\frac{1}{2}\right)} = \frac{8}{\sqrt{5}}s$$ **10 A (3)**

7. (From system ABC)



1. $R+S=5w$

Let $|AB|=|AC|=2a$

$\curvearrowright C:(3w)3a\cos\sigma+2wa\cos\sigma=4Ra\cos\sigma$

$\Rightarrow R=\frac{11w}{4}, S=\frac{9w}{4}$

Diagram: **10 A (3)**
Equations: **10 A (3)**
Solution: **10 A (3)**

Rod $AB: R+Y=3w \Rightarrow y=\frac{w}{4}$

$\curvearrowright B: y2a\cos\sigma-(3w)a\cos\sigma+x2a\sin\sigma=0$

$\Rightarrow \frac{w}{2}\cos\sigma-3w\cos\sigma+2x\sin\sigma=0$

$\Rightarrow 2x\sin\sigma=\frac{5w}{2}\cos\sigma$

$\Rightarrow x\tan\sigma=\frac{5w}{4}$

$\therefore x=\frac{5w}{4\tan\sigma}$

10 A (3)

At $B: m_1=\frac{11w}{4}/-\frac{5w}{4\tan\sigma}=\frac{-11\tan\sigma}{5}$

At $C: m_2=\frac{9w}{4}/\frac{5w}{4\tan\sigma}=\frac{9\tan\sigma}{5}$

Perp $\Rightarrow m_1.m_2=-1 \Rightarrow \frac{-99\tan^2\sigma}{25}=-1$

$\Rightarrow \tan\sigma=\frac{5}{3\sqrt{11}}$

10 A (3)

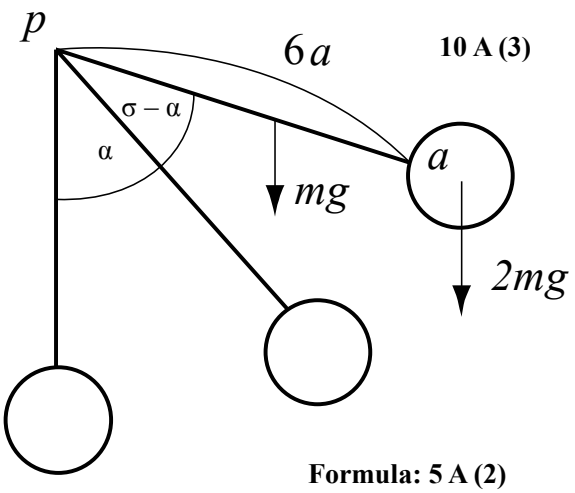
8.

(i) **Formula: 5 A (2)**

$$I_{system} = I_p(rod) + I_p(disc)$$

$$= \frac{4}{3}m(3a)^2 + \frac{1}{2}2m(a)^2 + 2m(7a)^2$$

$$12ma^2 + ma^2 + 98ma^2 = 111ma^2$$



10 A (3)

$$\curvearrowright p: 3mgh = mg(3a) + 2mg(7a) = 17mga$$

$$\Rightarrow h = \frac{17a}{3}$$

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{111ma^2}{3mg\left(\frac{17a}{3}\right)}} = 2\pi\sqrt{\frac{111a}{17g}}$$

Formula: 5 A (2)

10 A (3)

(ii) $\frac{l}{g} = \frac{111a}{17g} \Rightarrow l = \frac{111a}{17}$ **5 A (2)**

(iii) $P.E. = K.E.$

$$= mg(3a\cos\alpha - 3a\cos\sigma) + 2mg(7a)(\cos\alpha - \cos\sigma) = \frac{1}{2}(111)ma^2(w)^2$$

$$\Rightarrow 17\cos\alpha - 17\cos\sigma = \frac{(111)a(w)^2}{2g}$$

$$\Rightarrow w^2 = \frac{34g(\cos\alpha - \cos\sigma)}{111a}$$

$$\Rightarrow w = \sqrt{\frac{34g(\cos\alpha - \cos\sigma)}{111a}}$$

15 A (5)

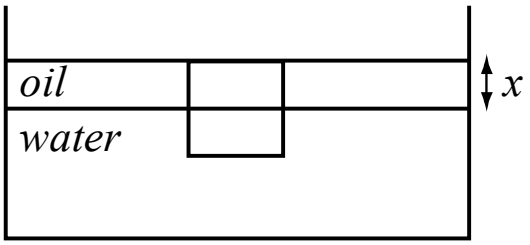
9.

(a) $B = \dfrac{w}{s}$

$\dfrac{17w(1)}{25} = w = s = \dfrac{17}{25}$

$B_w + B_{oil} = w$

5 A (2)



5 A (2)

$1000A(25-x)g + 600Axg = 680A(25)g$

$\Rightarrow 25000 - 1000x + 600x = 17000$

$\Rightarrow 400x = 8000$

$\therefore x = 20\text{ cm}$

X: depth of layer of oil

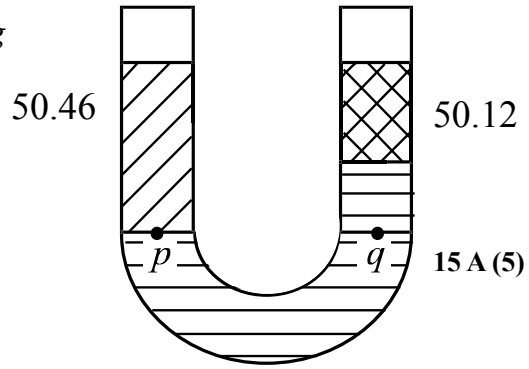
A: Cross – sectional area

10 A (3)

(b) (i) Pressure at p = Pressure at q

$50.64(1000)g = 50.12pq + 0.34(13,600)g$

$p = 918.12$



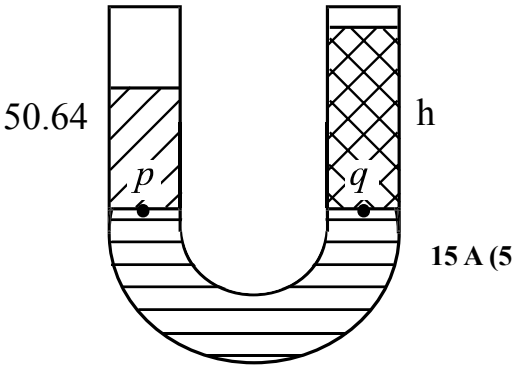
15 A (5)

(ii) Pressure at p = Pressure at q

$50.64(1000)g = h(918.12)g$

$\Rightarrow h = 55.2\text{ cm}$

$= 552\text{ mm}$



15 A (5

10.

(a) $\dfrac{\partial y}{\partial x} = 2\cos 2x + \sin 4x$

$\int dy = \int (2\cos 2x + \sin 4x)dx$

$\Rightarrow y = \sin 2x - \dfrac{1}{4}\cos 4x + c$ **10 A (3)**

$y = 1$ when $x = \dfrac{\pi}{4} \Rightarrow 1 = 1 + \dfrac{1}{4} + c \Rightarrow c = -\dfrac{1}{4}$

$y = \sin 2x - \dfrac{1}{4}\cos 4x - \dfrac{1}{4}$ **10 A (3)**

$x = \dfrac{\pi}{2} \Rightarrow y = -\dfrac{1}{2}$ **5 A (2)**

(b) $v\dfrac{dv}{ds} = -kv^2$

$\Rightarrow \int_8^4 \dfrac{dv}{v} = -k\int_0^{35} ds$

$\ln v\big|_8^4 = -|ks|_0^{35}$

$\Rightarrow \ln 2 = 35k$ **10 A (3)**

$\dfrac{dv}{dt} = -kv^2$

$\int_8^4 \dfrac{dv}{v^2} = -k\int_0^t dt$

$\Rightarrow -\dfrac{1}{v}\Big|_8^4 = -k|t|_0^t$

$\Rightarrow kt = \dfrac{1}{8}$

$\therefore A.speed = \dfrac{dist}{time} = \dfrac{35k}{kt} = \dfrac{\ln 2}{\frac{1}{8}}$

$= 8\ln 2$

15 A (5)

Ordinary Solutions

1.

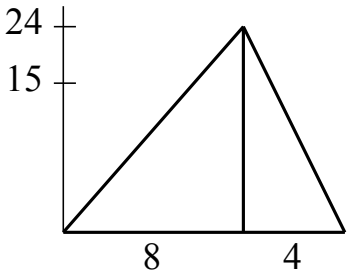


Diagram: **10 A (3)**

(i) $v = 0 + 8a = 24 \Rightarrow a = 3$ **10 A (3)**

(ii) $d = 6$ **10 A (3)**

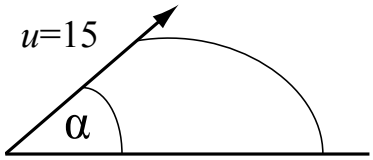
(iii) $A.S. = \dfrac{d}{t} = \dfrac{144}{12} = 12 \text{ m/s}$ **10 A (3)**

(iv) $15 = 0 + 3t \Rightarrow t_1 = 5s$

$15 = 24 - 6t = 0 \Rightarrow t_2 = 1\dfrac{1}{2}s$ **10 A (3)**

2. **(i)** $s_y = ut - \dfrac{1}{2}gt^2$

$= 15(2) - 5(2)^2 = 10 \text{ m}$ **10 A (3)**



(ii) $s_x = ut = 18 \Rightarrow 15t = 18 \Rightarrow t = 1\dfrac{1}{5}s$ **10 A (3)**

$v_x = 15, v_y = u - gt = 15 - 10\left(\dfrac{6}{5}\right) = 3$

$\tan\sigma = \dfrac{3}{15} \Rightarrow \sigma = 11.3° \Rightarrow E.11.3°N$ **10 A (3)**

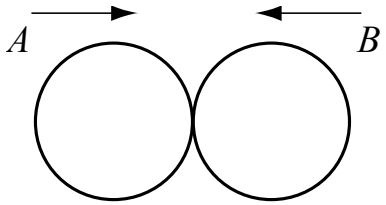
$\left|\vec{v}\right| = \sqrt{15^2 + 3^2} = 15.3 \text{ m/s}$ **10 A (3)**

(iii)

$Range = s_y = 0 \Rightarrow 15t - 5t^2 = 0 \Rightarrow t = 3s$

$s_x = 15(3) = 45 \text{ m}$ **10 A (3)**

3.



Before	Mass	After
$8\vec{i}$	3	$p\vec{i}$
$-6\vec{i}$	5	$q\vec{i}$

COM $3p+5q=24-30=-6$

NEL $\dfrac{p-q}{14}=-\dfrac{1}{3}\Rightarrow 3p-3q=-14$

$3p+5q=-6$
$3p-3q=-14$ **10 A (3)**
$8q=8$
$q=1$
$p=-\dfrac{11}{3}$

(i) $-\dfrac{11}{3}$ m/s, 1 m/s **10 A (3)**

(ii) $K.E._{Loss}=\dfrac{1}{2}(5)\left[6^2-1^2\right]=87.5$ J **20 A (6)**

(iii) $ChangeMom=3\left[8-\left(-\dfrac{11}{3}\right)\right]=35$ NS **10 A (3)**

4. $(\updownarrow)$ $R+60=300$
$\Rightarrow R=240$

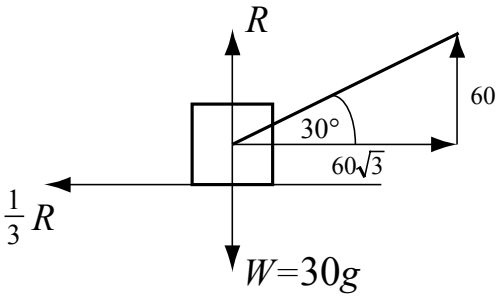


Diagram **10 A (3)**

(i) $60\sqrt{3}$ N, 60 N **10 A (3)**

(ii) R = 240 **10 A (3)**

(iii) $30a=60\sqrt{3}-80=23.92$
$\Rightarrow a=0.797\approx 0.8$ **10 A (3)**

(iv) **Work : FXS** $=23.92\times 1.5=35.88$ J **10 A (3)**

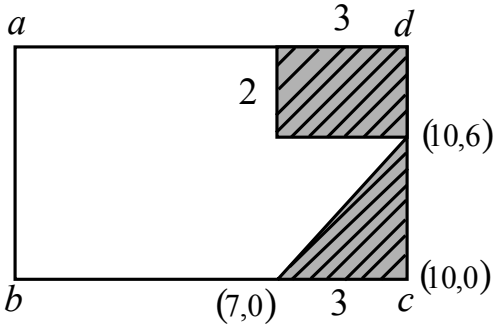
5. **(a)** $\left.\begin{array}{l} 3\text{ N at }(3,2) \\ 8\text{ N at }(4,-1) \\ 5\text{ N at }(-1,6) \\ 4\text{ N at }(2,-5) \end{array}\right\} 20\text{ N at }(x,y)$

$\curvearrowright x: 3(2)+8(-1)+5(6)+4(-5)=8=2oy \Rightarrow y=\frac{2}{5}$ **10 A (3)**

$\curvearrowright y: 3(3)+8(4)+5(-1)+4(2)=44=2ox \Rightarrow x=2\frac{1}{5}$ **10 A (3)**

Centre of gravity $\left(2\frac{1}{5},\frac{2}{5}\right)$ **5 A (2)**

(b)



$80\text{ at }(5,4)$

$6\text{ at }\left(8\frac{1}{2},7\right)$

$9\text{ at }(9,2)$

$65\text{ at }(x,y)$

$6\left(8\frac{1}{2}\right)+9(9)+65(x)=80(5)$

$51+81+65x=400 \Rightarrow x=\frac{268}{65}$ **10 A (3)**

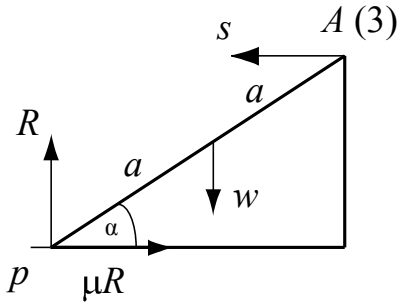
$6(7)+9(2)+65(y)=80(4)$

$42+18+65y=320 \Rightarrow y=4$ **10 A (3)**

Centre of gravity: $\left(\frac{268}{65},4\right)$ **5 A (2)**

6.

Diagram: 10 A (3)



$R = w = 15g$

10 A (3)

$S = 0.6R = 0.6w$

10 A (3)

$\curvearrowright p: \frac{3}{5}w \times 2a\sin\alpha = w \times a\cos\alpha$

10 A (3)

$\Rightarrow \tan\alpha = \frac{5}{6}$

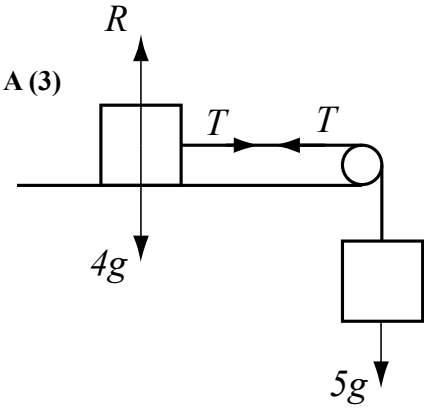
10 A (3)

7.

$5g - T = 5a, R = 4g, T = 4a$

$5g - 4a = 5a$

Diagram: 10 A (3)



(i) $\Rightarrow a = \frac{5g}{9}$

10 A (3)

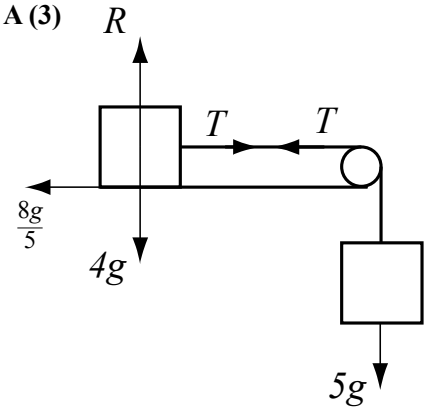
(ii) $T - \frac{2}{5}(4g) = 4a$

$\Rightarrow T = 4a + \frac{8g}{5}$

$5g - 4a - \frac{8g}{5} = 5a$

$\therefore a = \frac{17g}{45}$

Diagram: 10 A (3)



20 A (6)

8.

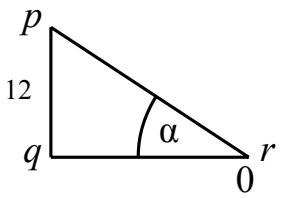
$|pr| = x \therefore |qr| = (18 - x)$

$\Rightarrow x^2 = (18-x)^2 + 12^2$

$x^2 = 324 - 36x + x^2 + 144$

$\therefore x = 13$

$\therefore \sin\alpha = \frac{12}{13}, \cos\alpha = \frac{5}{13}$



10 A (3)

Forces: **Resolved:**

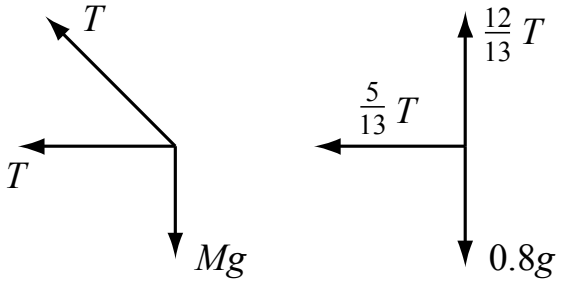


Diagram: 10 A (3)

1. $\frac{12}{13}T = 0.8g \Rightarrow T = \frac{10.4g}{12} = \frac{13g}{15}$ **10 A (3)**

2. $F_c = Mw^2 r = \left(T + \frac{5}{13}T\right) \Rightarrow Mw^2(5) = \frac{18T}{13}$ **15 A (5)**

$\Rightarrow w^2 = \frac{18T}{5M(13)} = \frac{18}{5\times 0.8\times 13} = \frac{3}{10}g$

$w = \sqrt{\frac{3}{10}g} = \sqrt{3}$ **5 A (2)**

9. **(i)** *Up thrust = Weight − Apparent weight*

$B = 0.078\times 10 - 0.06$

$= 0.72$ N **20 A (6)**

(ii) *Weight of water displaced* $= 0.72$

$pvg = 0.72$

$1000\times v\times 10 = 0.72$

$v = 7.2\times 10^{-5}\,\text{m}^3$ **20 A (6)**

$Density = \frac{Mass}{Volume}$

$= \frac{0.078}{7.2\times 10^{-5}}$ **10 A (3)**

$= \frac{13}{12}\times 10^3$ kg m^{-3}

Blank Page

Blank Page