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PRE-LEAVING CERTIFICATE EXAMINATION, 2007

APPLIED MATHEMATICS

MARKING SCHEME

HIGHER AND ORDINARY LEVEL

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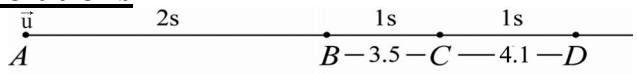
Higher Solutions

1. (a)



$\underline{AB}: S = 2u + \frac{1}{2} f(4) = 2u + 2f$ **(i)** **5m (Att. 2)**

$\underline{AC}: S + 3.5 = 3u + \frac{1}{2} f(9) = 3u + \frac{9}{2} f$ **(ii)** **5m (Att. 2)**

$\underline{AD}: S + 7.6 = 4u + \frac{1}{2} f(16) = 4u + 8f$ **(iii)** **5m (Att. 2)**

$$\left.\begin{array}{l} \textbf{(ii)} - \textbf{(i)}: u + \frac{5}{2} f = 3.5 \\ \textbf{(iii)} - \textbf{(ii)}: u + \frac{7}{2} f = 4.1 \end{array}\right\} \begin{array}{l} f = 0.6 \text{ m/s}^2 \\ u = 2 \text{ m/s} \end{array}$$ **10m (Att. 2)**

(b) (i) $Sp = 54(t+2) - 4.9(t+2)^2 = -4.9t^2 + 34.4t + 88.4$

$Sq = 47.4t - 4.9t^2$

at collision $Sp = Sq \Rightarrow 13t = 88.4 \Rightarrow t = 6.8 \text{ s}$ **15m (Att. 5)**

(ii) $S = 47.4(6.8) - 4.9(6.8)^2 = 95.744 \text{ m}$ **10m (Att. 3)**

2. (a) $Sx = u\cos\alpha(5) = 60 \Rightarrow 5u\cos\alpha = 60$ **10m (Att. 3)**

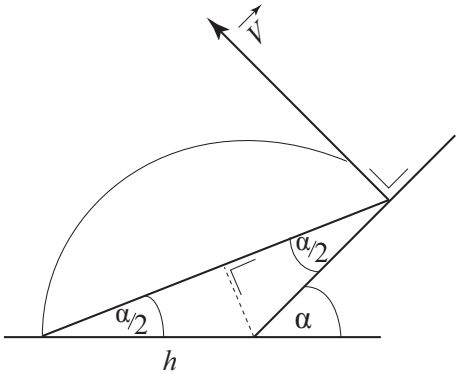
$Sy = 5u\sin\alpha - \frac{g}{2}(25) = 57.5 \Rightarrow 5u\sin\alpha = 180$ **10m (Att. 3)**

$\therefore \frac{5u\sin\alpha}{5u\cos\alpha} = \frac{180}{60} = 3 = \tan\alpha \Rightarrow u = 12\sqrt{10} \text{ m/s}$ **5m (Att. 2)**

(b) $Sy = v\sin\left(90° - \frac{\alpha}{2}\right)t - \frac{g}{2}\cos\frac{\alpha}{2}t^2 = 0$ **5m (Att. 2)**

$= v\cos\frac{\alpha}{2}t - \frac{g}{2}\cos\frac{\alpha}{2}t^2 = 0$ **5m (Att. 3)**

$\Rightarrow t = \frac{2v}{g}$ **10m (Att. 3)**



$Sx = v\sin\frac{\alpha}{2}\left(\frac{2v}{g}\right) + \frac{g}{2}\sin\frac{\alpha}{2}\left(\frac{2v}{g}\right)^2$

$= \frac{4v^2\sin\frac{\alpha}{2}}{g}$

$\Rightarrow \frac{4v^2\sin\frac{\alpha}{2}}{g} = 2h\cos\frac{\alpha}{2}$

$\Rightarrow 2v^2 = gh\cot\frac{\alpha}{2}$ **5m (Att. 2)**

3. **(a)** $\vec{V}_S = -10\vec{i},\ \vec{V}_W = a\vec{i} + b\vec{j},\quad \vec{V}_{WS} = (a+10)\vec{i} + b\vec{j}$

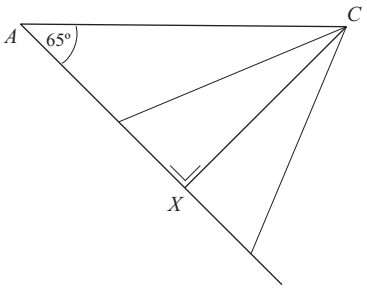
from $S\tan^{-1}\dfrac{1}{3}E \Rightarrow \dfrac{b}{a+10} = -3 \Rightarrow \boxed{3a+b=-30}$ **10m (Att. 3)**

$\vec{V}_S = -15\vec{i},\quad \vec{V}_{WS} = (a+15)\vec{i} + b\vec{j}$

from $S\tan^{-1}\dfrac{1}{4}E \Rightarrow \dfrac{b}{a+15} = -4 \Rightarrow \boxed{4a+b=-60}$ **10m (Att. 3)**

$\Rightarrow a = -30,\quad b = 60$

$\Rightarrow 30\sqrt{5}$ km/hr **from** $S\tan^{-1}\dfrac{1}{2}E$



5m (Att. 2)

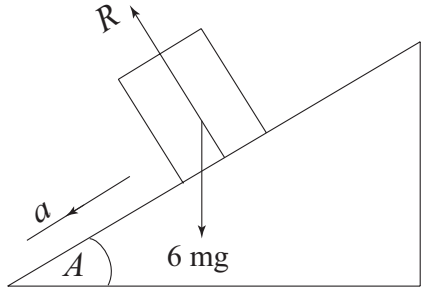
(b) $|CX| = 150\sin 65° = 135.95 \approx 136$

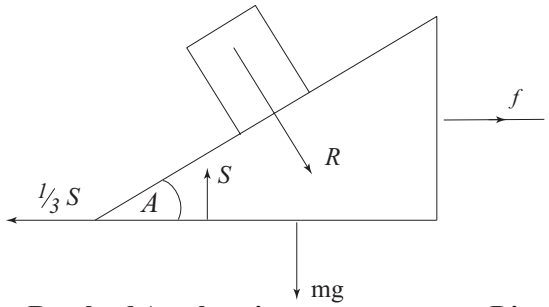
(i) **shortest distance:** 136 km **15m (Att. 5)**

(ii) $\sqrt{190^2 - 136^2} = 132.68\,\text{km}$

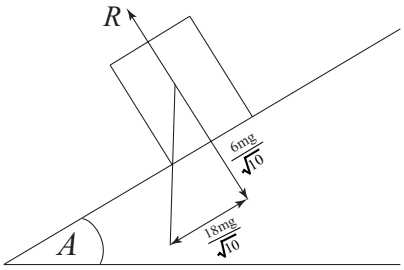
$\therefore \text{Time} = \dfrac{2\times 132.68}{200} = 1.3268 \approx 1\,\text{hr}.20\,\text{min}.$ **10m (Att. 3)**

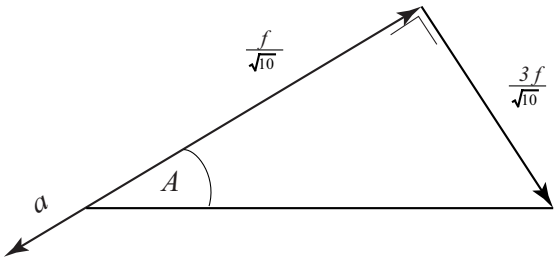
4. **Forces on Particle:** **Forces on Wedge:** **Diagrams:** **10m (Att. 3)**





Resolved Forces on the Particle: **Resolved Accelerations:** **Diagrams:** **10m (Att. 3)**





Equations:

$(\leftrightarrow)\quad \dfrac{18mg}{\sqrt{10}} = 6m\left(a - \dfrac{f}{\sqrt{10}}\right) \Rightarrow 3g = \sqrt{10}a - f$ **(i)** **5m (Att. 2)**

$(\updownarrow)\quad \dfrac{6mg}{\sqrt{10}} - R = 6m\left(\dfrac{3f}{\sqrt{10}}\right) \Rightarrow \sqrt{10}R = 6mg - 18mf$ **(ii)** **5m (Att. 2)**

$(\leftrightarrow)\quad \dfrac{3R}{\sqrt{10}} - \dfrac{S}{3} = mf \Rightarrow 9R - \sqrt{10}S = 3\sqrt{10}mf$ **(iii)** **5m (Att. 2)**

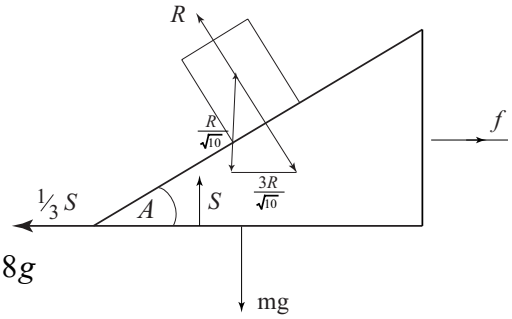
$(\updownarrow)\quad \dfrac{R}{\sqrt{10}} + mg = S$ **(iv)** **5m (Att. 2)**

(iv) into (iii) $\Rightarrow 9R - R - \sqrt{10}mg = 3\sqrt{10}mf$

$\Rightarrow 8R = \sqrt{10}m(g + 3f)$

$\Rightarrow R = \dfrac{\sqrt{10}m}{8}(g + 3f)$

Resolved Forces on the Wedge



into (ii) $\Rightarrow 10m(g + 3f) = 48mg - 144mf \Rightarrow 174f = 38g$

$\Rightarrow f = \dfrac{19g}{87}$ **10m (Att. 3)**

5. (a)

Before	Mass	After
V	m	p
W	m	q

$PCM : p + q = (V + W)$

$NLE : p - q = -e(V - W) = e(W - V)$

$P = \dfrac{1}{2}[V(1-e) + w(1+e)]$ **15m (Att. 5)**

$I = \dfrac{m}{2}[V(1-e) + W(1+e)] - mV$

$= \dfrac{m}{2}[V - ev + W + eW - 2V]$

$= \dfrac{m}{2}[-V - eV + W + eW]$

$= \dfrac{m}{2}[(1+e)(W - V)]$

$\Rightarrow |I| = \dfrac{m}{2}[(1+e)(V - W)]$ **10m (Att. 3)**

(b)

Before	Mass	After
$8\vec{i} + 3\vec{j}$	3 kg	$p\vec{i} + 3\vec{j}$
$3\vec{i} - 2\vec{j}$	5 kg	$q\vec{i} - 2\vec{j}$

$PCM : 3p + 5q = 39$

$NLE : p - q = -3$

10m (Att. 3)

10m (Att. 3)

(i) $\vec{V}_A = 3\vec{i} + 3\vec{j}$ **(ii)** $\vec{V}_B = 6\vec{i} - 2\vec{j}$ **5m (Att. 2)**

6. **(i)** **at** $B: v^2 = w^2\left(16a^2 - a^2\right) = 15a^2w^2 \Rightarrow v_1 = \sqrt{15}aw$

at $C: v^2 = w^2\left(16a^2 - y^2\right) \Rightarrow v_2 = w\sqrt{16a^2 - y^2}$

$$\Rightarrow \sqrt{16a^2 - y^2} = \frac{1}{2}\sqrt{15}a$$

$$\Rightarrow 16a^2 - y^2 = \frac{15a^2}{4} \Rightarrow y = \frac{7a}{2}$$

acc at $B: -w^2a$

acc at $C: -\frac{7}{2}w^2a \qquad \therefore B:C = 1:3\frac{1}{2}$ **20m (Att. 6)**

(ii) $x = A\sin wt$

at $B: a = 4a\sin wt \Rightarrow \sin wt = \frac{1}{4} \Rightarrow wt = 0.25$

at $C: 3.5a = 4a\sin w(t+1) \Rightarrow \sin w(t+1) = \frac{7}{8} \Rightarrow w(t+1) = 1.065$

$\therefore w = 0.815 \qquad \Rightarrow$ P. Time = 7.7 s **20m (Att. 6)**

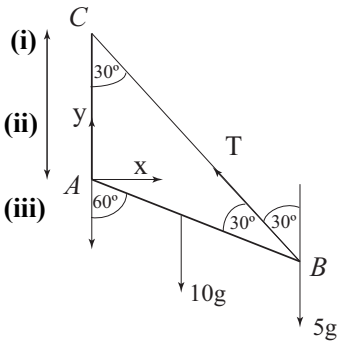
(iii) $V_{MAX} = 3.26a$ m/s; $A_{MAX} = 2.66a$ m/s^2 **10m (Att. 3)**

7. **(a)** $\widehat{A}: T.6\sin 30° - 10g.3\sin 60° - 5g.6\sin 60° = 0$ **(i)**

$\widehat{C}: 6X - 10g.3\sin 60° - 5g.6\sin 60° = 0$ **(ii)**

$(\updownarrow): y + T\cos 30° - 10g - 5g = 0$ **(iii)**

$\Rightarrow T = 10\sqrt{3}g$, $x = 5\sqrt{3}g$, $y = 0$ **25m (Att. 8)**



(b) **let** $|AB| = l$

$(\updownarrow) R + \mu s = w \qquad (\leftrightarrow) S = \psi R$

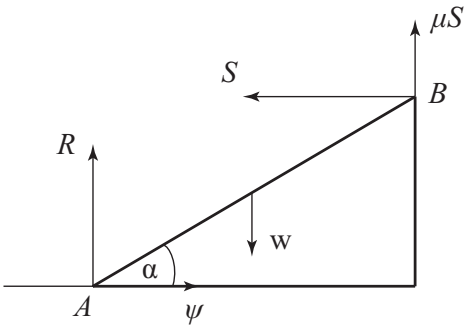
$\widehat{A}: \; S(l\sin\alpha) + R(0) + \mu s(l\cos\alpha) = w\left(\frac{1}{2}l\cos\alpha\right)$

$$\Rightarrow S(\sin\alpha) + \mu s\cos\alpha = \frac{w}{2}\cos\alpha$$

$$\Rightarrow S\tan\alpha + \mu s = \frac{w}{2} = \frac{1}{2}R + \frac{1}{2}\mu s$$

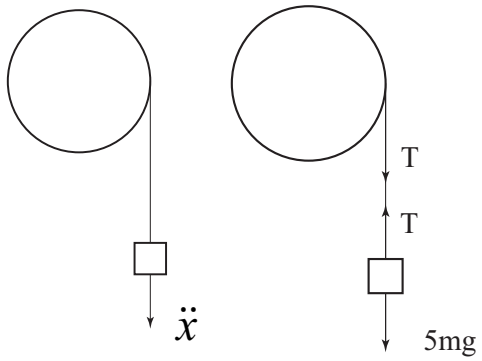
$$\Rightarrow S\tan\alpha + \frac{1}{2}\mu s = \frac{1}{2}R = \frac{1}{2}\frac{S}{\psi}$$

$$\Rightarrow 2\psi\tan\alpha + \psi\mu = 1 \Rightarrow \tan\alpha = \frac{1-\psi\mu}{2\psi}$$ **25m (Att. 8)**



8. (a) Text. 20m (Att. 8) **Accelerations** **Forces**

(b) 5m mass: $5mg - T = 5m\ddot{x},\ x = r\theta$



Disc: $L = r\ddot{\theta} \Rightarrow Tr = \left(\frac{1}{2}mr^2\right)\ddot{\theta}$

$$\Rightarrow T = \frac{1}{2}m\ddot{x}$$

(i) & (ii) $$\Rightarrow 5mg - \frac{1}{2}m\ddot{x} = 5m\ddot{x}$$

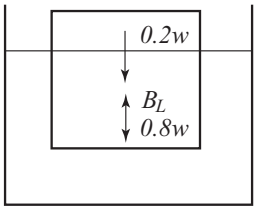
$$\Rightarrow \ddot{x} = \frac{10g}{11},\ T = \frac{5mg}{11}$$ **10m (Att. 3)**

(iii) $$S = \frac{1}{2}\times\frac{10g}{11}t^2 = \frac{5g}{11}t^2$$ **10m (Att. 3)**

(iv) $$2\pi\sqrt{\frac{5}{11}t^2} = 6 \Rightarrow t = 1.4s$$ **10m (Att. 3)**

9. (a) Text **10m (Att. 3)**

(i) $B_L = S_L B_W = S_L\left(\frac{w}{s}\right) = 1.2\left(\frac{0.8w}{s}\right) = \frac{0.96w}{s}$

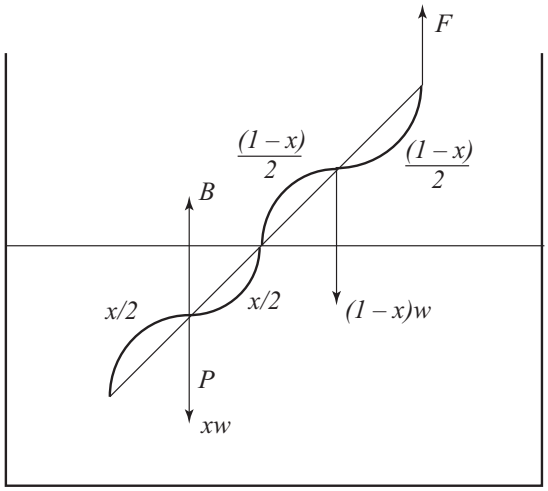


Since in equilibrium $0.2w + 0.8w = B_L = \frac{0.96w}{s} = w$

$\Rightarrow$ **(i)** $S = 0.96$ **10m (Att. 3)**

(ii) $\frac{24}{25}$ **5m (Att. 2)**

(b) $B_W = \frac{w}{S} = \frac{xw}{\frac{5}{9}} = \frac{9xw}{5}$



$$\frac{9xw}{5} + F = w \quad \textbf{(i)}$$

$$\widehat{P}: (1-x)w\left(\frac{x}{2} + \frac{1-x}{2}\right) = F\left(\frac{x}{2} + 1 - x\right)$$

$$\Rightarrow \frac{w}{2}(1-x) = F\left(1 - \frac{x}{2}\right)$$

$$\Rightarrow F = \frac{w(1-x)}{2-x} \Rightarrow \frac{9xw}{5} + \frac{w(1-x)}{2-x} = w$$

$$\Rightarrow 9x^2 - 18x + 5 = 0 \Rightarrow (3x-1)(3x-5) = 0 \Rightarrow x = \frac{1}{3}, \frac{5}{3}$$

(i) $\frac{2l}{3}$ **20m (Att. 6)**

(ii) $F = \frac{2w}{5}$ **5m (Att. 2)**

10. (a) $\int \frac{4x dx}{6+x^2} = \int \frac{dy}{1+y}$

$\Rightarrow 2\ln(6+x^2) = \ln(1+y) + c$

$x = 3, y = 2 \Rightarrow 2\ln 15 = \ln 3 + c \Rightarrow c = \ln 75$

$\therefore 2\ln(6+x^2) = \ln(1+y) + \ln 75$

$\Rightarrow \ln \frac{(x^2+6)^2}{75} = \ln(1+y)$

$\Rightarrow y = \frac{(x^2+6)^2}{75} - 1$ **25m (Att. 8)**

(b) $-ma = m(0.098v^2) + mg$

$\Rightarrow a = -0.098v^2 - g \Rightarrow \frac{dv}{dt} = -0.98v^2 - g$

$\Rightarrow \frac{dv}{dt} = -0.98(v^2 + 10^2)$

$\Rightarrow \int \frac{dv}{v^2+10^2} = -0.098 \int dt$

$\Rightarrow \frac{1}{10}\tan^{-1}\frac{v}{10} = -0.098t + c \Rightarrow c = \frac{1}{10}\tan^{-1} 15 \quad (v = 150,\ t = 0)$

$\Rightarrow \frac{1}{10}\tan^{-1}\frac{v}{10} = -0.098t + \frac{1}{10}\tan^{-1} 15$

at gt. ht $v = 0 \Rightarrow 0.098t = \frac{1}{10}\tan^{-1} 15$

$\Rightarrow t = 1.5\text{ s}$ **25m (Att. 8)**

Ordinary Solutions

1. **(i)**

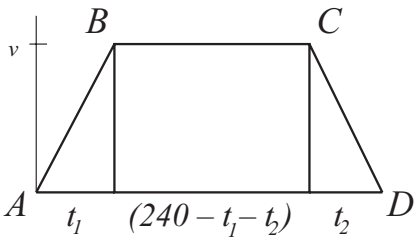


Diagram: 10 m(Att. 3)

(ii) $\underline{AB}: \frac{1}{2}t_1 v = 250 \Rightarrow t_1 = \frac{500}{v}$

$\underline{BC}: v(240 - t_1 - t_2) = 1500$

$\underline{CD}: \frac{1}{2}t_2 v = 500 \Rightarrow t_2 = \frac{1000}{v}$

$\Rightarrow v = 12\frac{1}{2}\text{ m/s} \Rightarrow t_1 = 40\text{ s},\ t_2 = 80\text{ s}$

$\therefore a = \frac{5}{16}\text{ m/s}^2$ **20m (Att. 6)**

(iii) $f = \frac{5}{32}\text{ m/s}^2$ **10m (Att. 3)**

(iv) **A. Speed** $= \frac{2250}{240} = 9.375\text{ m/min}$ **10m (Att. 3)**

2. **(a)** $\vec{u} = 24\vec{i} + 18\vec{j}$

(i) $Sx = 24t = 72 \Rightarrow t = 3s \Rightarrow Sy = 18\times 3 - 5\times 9 = 9\text{ m}$ **10m (Att. 3)**

(ii) $Sy = 0 \Rightarrow 18t - 5t^2 = 0 \Rightarrow t = 3.6s \Rightarrow Sx = 24\times 3.6 = 86.4\text{ m}$

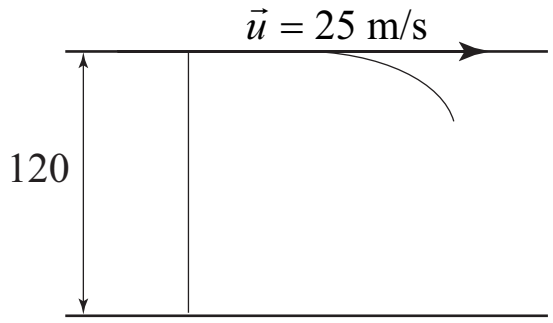
Past tree: 14.4 m **15m (Att. 5)**

(b) $\vec{u} = 25\vec{i}$

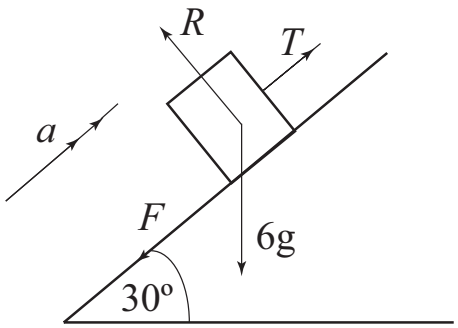
(i) $Sy = -\frac{1}{2}\times 10\times t^2 = -120$

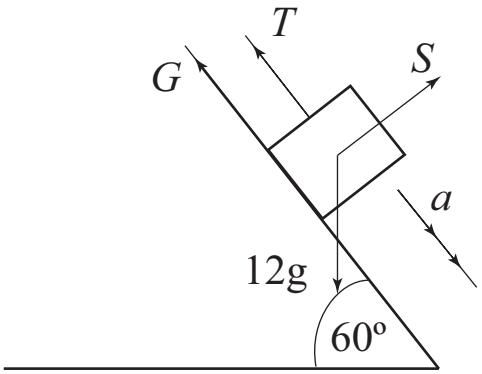
$\Rightarrow t = 4.9\text{ s}$ **15m (Att. 2)**

(ii) $Sx = 25\times 4.9 = 122.5\text{ m}$ **10m (Att. 2)**



3. **Diagrams: 10m, 10m (Att. 3, Att. 3)**





$(\parallel)$ $T - F - 3g = 6a$ $(\parallel)$ $6\sqrt{3}g - T - G = 12a$ **Equations: 20m (Att. 6)**

$(\perp)$ $R = 3\sqrt{3}g$ $(\perp)$ $S = 6g$

also $F = \sqrt{3}g$ **also** $G = 2g$

$$\left.\begin{aligned} T - \sqrt{3}g - 3g &= 6a \\ -T - 2g + 6\sqrt{3}g &= 12a \end{aligned}\right\} \quad \begin{aligned} a &= 2.03\ \text{m/s}^2 \\ T &= 59.5\ \text{N} \end{aligned}$$

10m (Att. 3)

4. **(a)** **(i)** $\vec{V}s = 7\vec{j}$ $\vec{V}ps = 15\vec{i} - 15\vec{j}$ **10m (Att. 4)**

(ii) $\vec{V}p = 15\vec{i} - 8\vec{j}$ **5m (Att. 2)**

(iii) $\left|\vec{V}p\right| = 17\ \text{km/hr}$ **5m (Att. 2)**

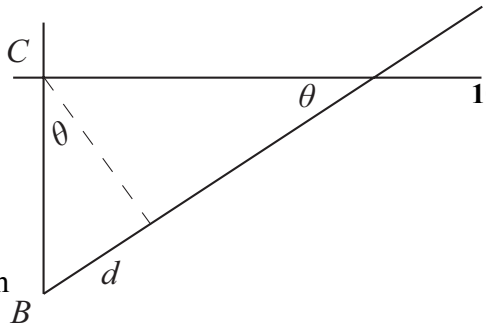
(iv) **Direction:** $\tan\theta = \dfrac{8}{15}$, $\theta = 28° \Rightarrow E28°S$ **5m (Att. 2)**

(b) $\vec{V}_B = 30\vec{i}$, $\vec{V}_C = 9\vec{i} - 9\sqrt{3}\vec{j}$

(i) $\vec{V}_{BC} = 21\vec{i} + 9\sqrt{3}\vec{j}$ **10m (Att. 3)**

$\left|\vec{V}_{BC}\right| = 26.15\ \text{km/hr}$

$\tan\theta = \dfrac{3\sqrt{3}}{7} \Rightarrow d = 40 \times \sin\theta = 23.84\ \text{km}$



$\textbf{Time} = \dfrac{23.84}{26.15} = 0.911 = 55\,\text{min}$

(ii) **Time nearest each other at:** 3:55 **15m (Att. 5)**

5.

	Before	Mass	After
A:	6	5 kg	P
B:	0	3 kg	q

$PCM: 5p+3q=30$

$NLE: p-q=-3$

$p=\frac{21}{8}, q=\frac{45}{8}$

(i) $\vec{V}_A=\frac{21}{8}\vec{i}+0\vec{j},\ \vec{V}_B=\frac{45}{8}\vec{i}+0\vec{j}$ **20m (Att. 6)**

(ii) K.E Before: $\frac{1}{2}(5)(6)^2+\frac{1}{2}(3)(0)^2=90$

K.E After: $\frac{1}{2}(5)\left(\frac{21}{8}\right)^2+\frac{1}{2}(3)\left(\frac{45}{8}\right)^2=64\frac{11}{16}$

Loss: $25\frac{5}{16}$ J **20m (Att. 6)**

(iii) $v=u+at\Rightarrow 0=\frac{45}{8}+3a\Rightarrow a=-\frac{15}{8}$

Force: $F=Ma\Rightarrow F=3\left(\frac{-15}{8}\right)=\frac{-45}{8}$ N

$v^2=u^2+2as\Rightarrow\left(\frac{45}{8}\right)^2=2\left(\frac{15}{8}\right)s\Rightarrow s=8\frac{7}{16}$ m

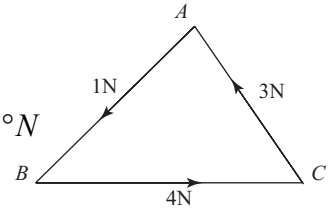
$w=F\times s=\frac{45}{8}\times 8\frac{7}{16}=47\frac{59}{128}(47.46)$

Power $=\frac{Work}{Time}=\frac{47.46}{3}=15.82$ Watts **10m (Att. 3)**

6. (a) (i) $\left.\begin{array}{l}(\leftrightarrow)\ \ 4-3\cos 60°-1\cos 60°=2\\(\updownarrow)\ \ 3\sin 60°-1\sin 60°=\sqrt{3}\end{array}\right\}\ \text{Res}=2\vec{i}+\sqrt{3}\vec{j}$ **10m (Att. 3)**

(ii) Magn: $\sqrt{4+3}=\sqrt{7}$ N

Dir: $\tan\theta=\frac{\sqrt{3}}{2}\Rightarrow\theta=41°$ to $BC\Rightarrow E41°N$



10m (Att. 3)

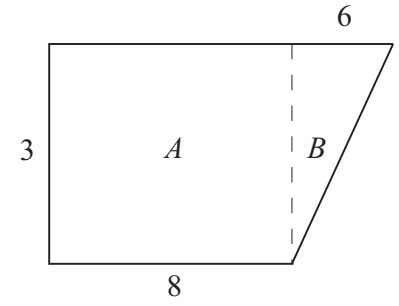
(iii) $\widehat{B}$ **Sum of moments = moment of resultant**

$1\times 0+4\times 0+3\times 2\sin 60°=\sqrt{7}d$

$\Rightarrow d=3\sqrt{\frac{3}{7}}$m **10m (Att. 3)**

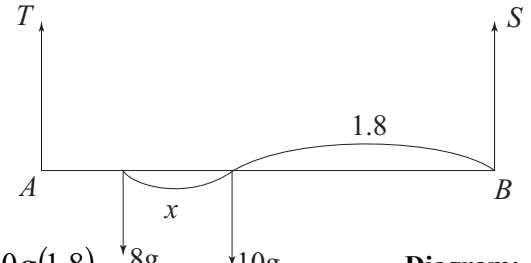
(b) Area: $A:24$ **at** $\left(4,1\frac{1}{2}\right)$

Area: $B:9$ **at** $(10,2)$



$\left.\begin{array}{l}M(y):24(4)+9(10)=186=33x\Rightarrow x=5\frac{7}{11}\\M(x):24\left(1\frac{1}{2}\right)+9(2)=54=33y\Rightarrow y=1\frac{7}{11}\end{array}\right\}\ \left(5\frac{7}{11},1\frac{7}{11}\right)$ **20m (Att. 6)**

7. (a) (i)



8g placed *x* cm from *CG*
Then *T* = 100 = 10 g

$\widehat{B}: 10g \times 3.6 = 8g(1.8+x) + 10g(1.8)$ **Diagram: 10m (Att. 3)**

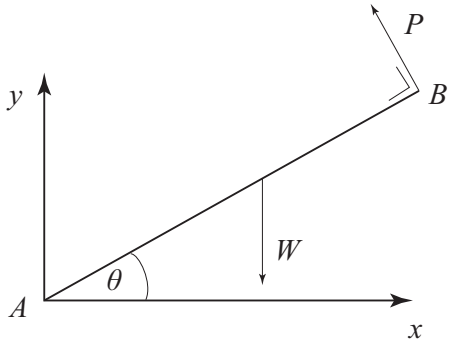
(ii) $\Rightarrow 8x + 14.4 = 18$ **10m (Att. 3)**

(iii) $\therefore x = 0.45$ **5m (Att. 2)**

(b) $(\updownarrow)$ $y + p\cos\theta = w$

$(\leftrightarrow)$ $x = p\sin\theta$

$\widehat{A}:$ $wl\cos\theta = 2lp \Rightarrow p = \dfrac{w\cos\theta}{2}$



$\Rightarrow y + \dfrac{w\cos^2\theta}{2} = w$

$\Rightarrow 2y + w\cos^2\theta = 2w$

$2y = w(2 - \cos^2\theta)$ **25m (Att. 8)**

8. (a) $(\updownarrow)$ $R = 30g$

$(\leftrightarrow)$ $F = \mu R = 30\mu g$

$F = \dfrac{Mv^2}{r} = \dfrac{30\times 15^2}{50} = 30\mu g$

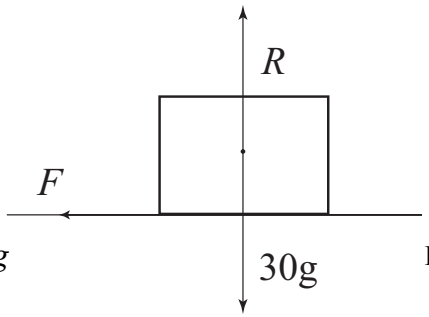
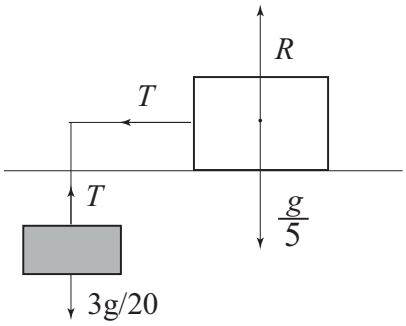


Diagram: 10m (Att. 3)

$\Rightarrow \mu = \dfrac{9}{20}$ **15m (Att. 5)**

(b) $(\updownarrow)$ $T = \dfrac{3g}{20}$

$(\leftrightarrow)$ $T = Mrw^2$



$\dfrac{3g}{20} = \dfrac{g}{5} \times r \times \dfrac{16\pi^2}{9}$

$\Rightarrow r = 0.0427$

$\Rightarrow l = 18.0427 \approx 18.04$ cm

40 revs / min

$\dfrac{40\times 2\pi}{60}$ rads / sec

$= \dfrac{4\pi}{3} = w$

25m (Att. 8)

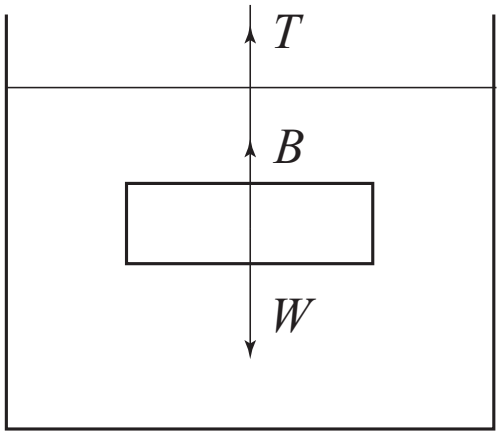
9. (a) Text **10m (Att. 3)**

Buoyancy = actual wt – apparent wt

$$B_W = 140 - 121 = 19 \text{ N}$$

$$B_L = 140 - 125 = 15 \text{ N}$$

$$= S_L B_W \qquad \Rightarrow S_L = \frac{B_L}{B_W} = \frac{15}{19}$$ **20m (Att. 6)**



(b) $v = \dfrac{M}{\rho} = \dfrac{14.5}{2500} = \dfrac{29}{5000} = 0.0058 \text{ m}^3$

$W = T + B$

$T = W - B$

$= 14.5 \text{ g} - 3.48 \text{ g}$

$= 11.02 \text{ g}$

$= 110.2 \text{ N}$

$M = V\rho = 0.0058 \times 600$

$= 3.48 \text{ kg}$

20m (Att. 6)