M32

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéim Mharcála	*Scrúduithe Ardteistiméireachta, 2006*
Matamaitic Fheidhmeach	*Ardleibhéal*
Marking Scheme	*Leaving Certificate Examination, 2006*
Applied Mathematics	*Higher Level*

General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2).

2 Mark all answers, including excess answers and repeated answers whether cancelled or not, and award the marks for the best answers.

3 Mark scripts in red unless candidate uses red. If a candidate uses red, mark the script in blue or black.

4 Number the grid on each script 1 to 10 in numerical order, not the order of answering.

5 Scrutinise **all** pages of the answer book.

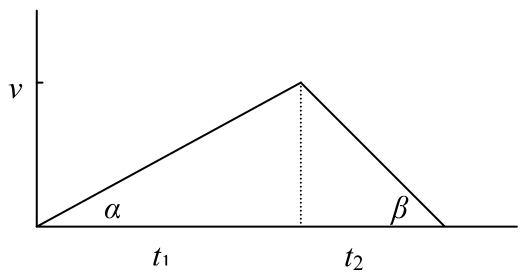
6 The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. **(a)** A lift starts from rest. For the first part of its descent it travels with uniform acceleration f. It then travels with uniform retardation $3f$ and comes to rest. The total distance travelled is d and the total time taken is t.

(i) Draw a speed-time graph for the motion.

(ii) Find d in terms of f and t.

(i)



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(ii)

$$\left.\begin{aligned} f &= \tan\alpha = \frac{v}{t_1} \\ 3f &= \tan\beta = \frac{v}{t_2} \end{aligned}\right\}$$ 5

$$3\left(\frac{v}{t_1}\right) = \frac{v}{t - t_1}$$

$$3t - 3t_1 = t_1$$

$$t_1 = \tfrac{3}{4}t$$ 5

$$d = \tfrac{1}{2}(t_1 + t_2)v \text{ or } \tfrac{1}{2}tv$$ 5

$$= \tfrac{1}{2}tv$$

$$= \tfrac{1}{2}tft_1$$

$$= \tfrac{3}{8}ft^2$$ 5 | 25

1 **(*b*)** Two trains P and Q, each of length 79.5 m, moving in opposite directions along parallel lines, meet at *o*, when their speeds are 15 m/s and 10 m/s respectively.
The acceleration of P is 0.3 m/s^2 and the acceleration of Q is 0.2 m/s^2.
It takes the trains *t* seconds to pass each other.

(i) Find the distance travelled by each train in *t* seconds.

(ii) Hence, or otherwise, calculate the value of *t*.

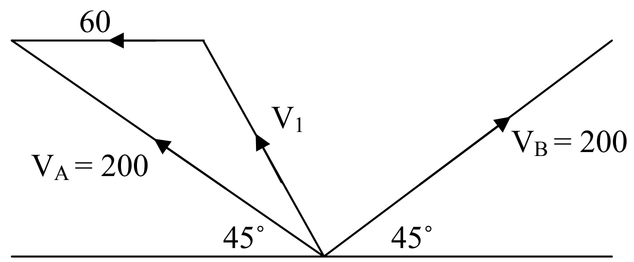
(iii) How long does it take for $\frac{2}{5}$ of the length of train Q to pass the point *o*?

(*i*) $S_P = ut + \frac{1}{2}at^2$ $= 15t + 0.15t^2$	5	
$S_Q = ut + \frac{1}{2}at^2$ $= 10t + 0.1t^2$	5	
(*ii*) $S_P + S_Q = 159$	5	
$15t + 0.15t^2 + 10t + 0.1t^2 = 159$ $t^2 + 100t - 636 = 0$ $(t-6)(t+106) = 0$ $\Rightarrow \quad t = 6 \text{ s}$	5	
(*iii*) $\frac{2}{5}(79.5) = 31.8$ $S_Q = 10t + 0.1t^2$ $31.8 = 10t + 0.1t^2$ $t^2 + 100t - 318 = 0$ $\Rightarrow \quad t = 3.0848 \text{ or } 3.1 \text{ s.}$	5	25

2. **(a)** Two aeroplanes A and B, moving horizontally, are travelling at 200 km/h relative to the ground. There is a wind blowing from the east at 60 km/h. The actual directions of flight of A and B are north-west and north-east respectively.

Find **(i)** the speed of aeroplane A in still air

(ii) the magnitude and direction of the velocity of A relative to B.



(i) $v_1^2 = 60^2 + 200^2 - 2(60)(200)\cos 45$ **5**

$= 26629.43725$

$v_1 = 163.2$ km/h **5**

(ii) $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ **5**

$= \left(-\frac{200}{\sqrt{2}}\vec{i} + \frac{200}{\sqrt{2}}\vec{j}\right) - \left(\frac{200}{\sqrt{2}}\vec{i} + \frac{200}{\sqrt{2}}\vec{j}\right)$

$= -\frac{400}{\sqrt{2}}\vec{i} + 0\,\vec{j}$ or $-200\sqrt{2}\,\vec{i} + 0\,\vec{j}$ **5**

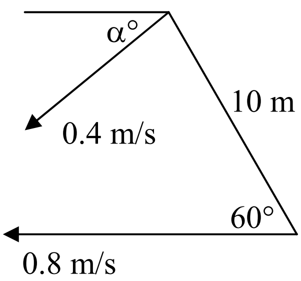
$\left.\begin{array}{ll}\text{magnitude:} & 200\sqrt{2} \text{ or } 282.84 \text{ km/h} \\ \text{direction:} & \text{west}\end{array}\right\}$ **5**

25

2 **(b)** A boy swims due west at a speed of 0.8 m/s.

A girl swims at 0.4 m/s in the direction $\alpha°$ south of west.

At a certain instant the girl is 10 m, $60°$ north of west of the boy and 10 s later she is due north of the boy.



(i) Find the distance travelled by the boy and the girl in 10 s.

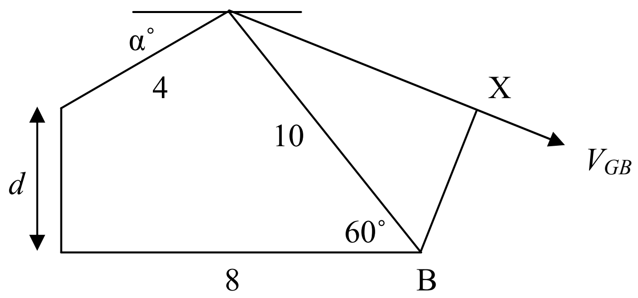
(ii) Hence, or otherwise, find the value of α .

(iii) Find the shortest distance between the boy and the girl in the subsequent motion.

(i) distance travelled by :

boy $= (0.8)(10) = 8$ m 5

girl $= (0.4)(10) = 4$ m 5



(ii) $8 = 10\cos 60 + 4\cos\alpha$

$\cos\alpha = \frac{3}{4}$ or $\alpha = 41.41°$ 5

(iii) $\vec{V}_{GB} = \vec{V}_G - \vec{V}_B$

$= (-0.3\ \vec{i} - 0.2646\ \vec{j}) - (-0.8\ \vec{i})$

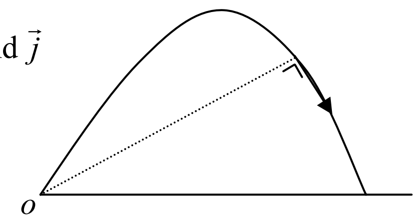
$= 0.5\ \vec{i} - 0.2646\ \vec{j}$

direction 27.89° south of east 5

Shortest distance $= \lvert BX \rvert$	Shortest distance
$= 10\sin(60 - 27.89)$	$= d\sin(90 - 27.89)$
$= 5.3$ m.	$= 6.01\sin(62.11) = 5.3$ m

5 25

3. **(a)** A particle is projected from a point o with velocity $9.8\vec{i} + 29.4\vec{j}$ m/s where $\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.



(i) Express the velocity and displacement of the particle after t seconds in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find, in terms of t, the direction in which the particle is moving after t seconds.

(iii) Find the two times when the direction of the particle is at right angles to the line joining the particle to o.

(i)

$$\vec{r} = (u\cos\alpha.t)\ \vec{i} \ + \ (u\sin\alpha.t - \tfrac{1}{2}gt^2)\ \vec{j}$$

$$\text{or} \ \ (9.8.t)\ \vec{i} \ + \ (29.4.t - \tfrac{1}{2}gt^2)\ \vec{j} \qquad 5$$

$$\vec{v} = (u\cos\alpha)\ \vec{i} \ + \ (u\sin\alpha - gt)\ \vec{j}$$

$$\text{or} \ \ (9.8)\ \vec{i} \ + \ (29.4 - gt)\ \vec{j} \qquad 5$$

(ii) direction:

$$\tan\theta = \frac{V_{\vec{j}}}{V_{\vec{i}}} = \frac{29.4 - gt}{9.8} \ \text{ or } \ 3 - t \qquad 5$$

(iii) direction of : line joining the particle to o

$$= \tan\phi = \frac{r_{\vec{j}}}{r_{\vec{i}}} = \frac{29.4t - \frac{1}{2}gt^2}{9.8t} \ \text{ or } \ 3 - \tfrac{1}{2}t$$

lines are perpendicular

$\Rightarrow$ product of slopes $= -1$

$$(3 - t)(3 - \tfrac{1}{2}t) = -1 \qquad 5$$

$$t^2 - 9t + 20 = 0$$

$$(t - 4)(t - 5) = 0$$

$$\Rightarrow \ t = 4\,\text{s} \ \text{ and } \ t = 5\,\text{s} \qquad 5 \qquad \boxed{25}$$

3 **(b)** A particle is projected up an inclined plane with initial velocity u m/s. The line of projection makes an angle $30°$ with the plane and the plane is inclined at $30°$ to the horizontal.
The plane of projection is vertical and contains the line of greatest slope.

Find, in terms of u, the range of the particle on the inclined plane.

$$r_j = 0$$ 5

$$u \sin 30.t \; - \; \tfrac{1}{2} g \cos 30.t^2 \; = \; 0$$ 5

$$\Rightarrow \quad t \; = \; \frac{2u \sin 30}{g \cos 30} \quad \text{or} \quad \frac{2u}{g\sqrt{3}}$$ 5

$$\text{Range} \; = \; u \cos 30.t \; - \; \tfrac{1}{2} g \sin 30.t^2$$ 5

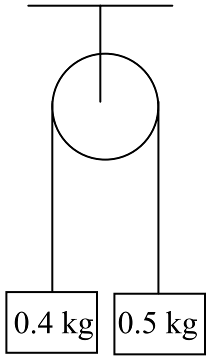
$$= u \cos 30.\left(\frac{2u}{g\sqrt{3}}\right) \; - \; \tfrac{1}{2} g \sin 30..\left(\frac{2u}{g\sqrt{3}}\right)^2$$

$$= \frac{u^2}{g} \; - \; \frac{u^2}{3g}$$

$$= \frac{2u^2}{3g}$$ 5 | 25

4. **(a)** Two particles of mass 0.4 kg and 0.5 kg are attached to the ends of a light inextensible string which passes over a fixed smooth light pulley.

The system is released from rest.



(i) Find the acceleration of the system, in terms of g.

(ii) After falling 1 m the 0.5 kg mass strikes a horizontal surface and is brought to rest. The string again becomes taut after t seconds. Find the value of t correct to two places of decimals.

(i)

$$0.5g - T = 0.5f \qquad 5$$

$$T - 0.4g = 0.4f \qquad 5$$

$$0.1g = 0.9f$$

$$f = \frac{g}{9} \text{ m/s}^2 \qquad 5$$

(ii)

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2\left(\frac{g}{9}\right)(1)$$

$$v = \sqrt{\frac{2g}{9}} \qquad 5$$

The 0.4 kg mass rises for $\frac{1}{2}t$ seconds before coming to instantaneous rest

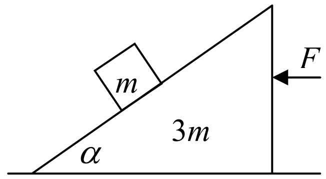
$$v = u + at$$

$$0 = \sqrt{\frac{2g}{9}} - g\left(\tfrac{1}{2}t\right)$$

$$t = 0.30 \text{ seconds} \qquad 5$$

25

4 **(b)** A smooth wedge of mass $3m$ and slope α rests on a smooth horizontal surface.
A particle of mass m is placed on the smooth inclined face of the wedge and is released from rest.
A horizontal force F is applied to the wedge to keep it from moving.



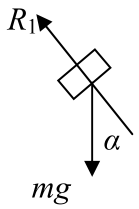
(i) Show, on separate diagrams, the forces acting on the wedge and on the particle.

(ii) Prove that the reaction between the wedge and the horizontal surface is

$$mg\left(3+\cos^2\alpha\right).$$

(iii) If the speed of the particle after 1 s is 4.9 m/s find the value of α.

(*i*)



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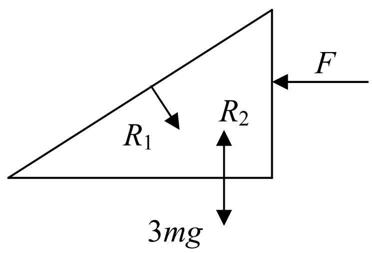

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(*ii*)

$$\left.\begin{aligned} R_1 &= mg\cos\alpha \\ R_2 &= 3mg + R_1\cos\alpha \end{aligned}\right\}$$

5

$$= 3mg + (mg\cos\alpha)\cos\alpha$$

$$= mg\left(3+\cos^2\alpha\right)$$

5

(*iii*)

$$mg\sin\alpha = mp$$

$$g\sin\alpha = p$$

$$v = u + at$$

$$4.9 = 0 + g\sin\alpha(1)$$

$$\sin\alpha = \frac{1}{2}$$

$$\alpha = 30^\circ$$

5

25

5. **(a)** A smooth sphere P, of mass 3 kg, moving with speed 6 m/s, collides directly with a smooth sphere Q, of mass 5 kg, which is moving in the same direction with speed 2 m/s. The coefficient of restitution for the collision is e.

(i) Find, in terms of e, the speed of each sphere after the collision.

(ii) If the loss of kinetic energy due to the collision is $k(1-e^2)$, find the value of k.

(i) PCM $\quad 3(6) + 5(2) = 3v_1 + 5v_2 \qquad$ 5

NEL $\quad v_1 - v_2 = -e(6-2) \qquad$ 5

$$\left.\begin{aligned} v_1 &= \frac{7-5e}{2} \\ v_2 &= \frac{7+3e}{2} \end{aligned}\right\}$$ 5

(ii) $\quad \text{KE before} = \frac{1}{2}(3)(6)^2 + \frac{1}{2}(5)(2)^2 = 64$

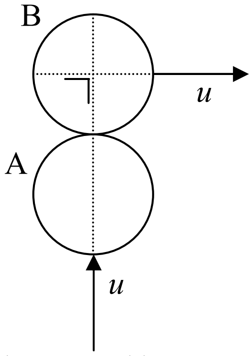
$\text{KE after} = \frac{1}{2}(3)(v_1)^2 + \frac{1}{2}(5)(v_2)^2$

$\text{Loss in KE} = 64 - \frac{3}{2}\left(\frac{7-5e}{2}\right)^2 - \frac{5}{2}\left(\frac{7+3e}{2}\right)^2 \qquad$ 5

$= 15(1\text{-}e^2)$

$\Rightarrow \quad k = 15 \qquad$ 5 $\qquad$ 25

5 **(*b*)** A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is moving in a perpendicular direction with the same speed u.



The line of centres at the instant of impact is perpendicular to the direction of motion of sphere B.

The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the speed of each sphere after impact and hence, or otherwise, show that it is not possible for the two spheres to have the same speed after impact.

(ii) Prove that $\tan\theta = \left(\dfrac{1+e}{2}\right)$, where θ is the angle through which sphere B is turned as a result of the impact.

(i) PCM $\quad m(u) + m(0) = mv_1 + mv_2$ 5

NEL $\quad v_1 - v_2 = -e(u-0)$ 5

$$\Rightarrow \quad v_1 = \frac{u}{2}(1-e) \quad \text{and} \quad v_2 = \frac{u}{2}(1+e)$$

Speed of A $= \dfrac{u}{2}(1-e)$

Speed of B $= \sqrt{u^2 + \dfrac{u^2}{4}(1+e)^2}$ or $\dfrac{u}{2}\sqrt{e^2+2e+5}$ 5

Speed of A = Speed of B

$$\frac{u}{2}(1-e) = \frac{u}{2}\sqrt{e^2+2e+5}$$

$\Rightarrow \quad e = -1$

$\Rightarrow$ The speeds can never be equal 5

(ii) Velocity of B after the collision is

$$u\vec{i} + \frac{u}{2}(1+e)\vec{j}$$

$$\tan\theta = \frac{\frac{u}{2}(1+e)}{u} = \frac{1}{2}(1+e)$$ 5

25

6. **(a)** A particle moves with simple harmonic motion of period 3π. At time t = 0, the particle passes through the centre of the oscillation. It passes through a point distant 4 m from the centre of motion with a speed of 5 m/s away from the centre.

Find, correct to two decimal places,

(i) the maximum acceleration of the particle

(ii) the time which elapses before it next passes through this point.

(i) Period $= \dfrac{2\pi}{\omega}$

$$3\pi = \frac{2\pi}{\omega} \quad \Rightarrow \quad \omega = \frac{2}{3}$$ 5

$$v = \omega\sqrt{a^2 - x^2}$$

$$5 = \tfrac{2}{3}\sqrt{a^2 - 16}$$

$$a = \sqrt{72.25} = 8.5$$ 5

max accel $= \omega^2 a$

$$= \left(\tfrac{2}{3}\right)^2(8.5) = 3.78\ \text{m/s}^2$$ 5

(ii)

$$x = a\cos\left(\omega.\tfrac{1}{2}t\right)$$

$$4 = 8.5\cos\left(\tfrac{1}{3}t\right)$$ 5

$$t = 3.24 \text{ seconds}$$ 5

or

$$x = a\sin(\omega.t)$$

$$4 = 8.5\sin\left(\tfrac{2}{3}t\right)$$

$$t = 0.7349$$ 5

required time $= 2\left(\tfrac{1}{4}\text{Period} - t\right)$

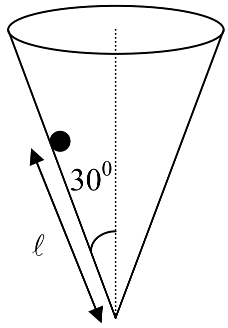
$$= 2\left(\frac{3\pi}{4} - 0.7349\right) = 3.24 \text{ s.}$$ 5

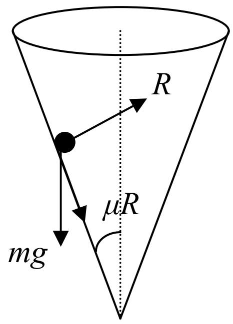
25

6 **(b)** A hollow cone with its vertex downwards and its axis vertical, revolves about its axis with a constant angular velocity of 4π rad/s.
A particle of mass m is placed on the inside rough surface of the cone. The particle remains at rest relative to the cone.

The coefficient of friction between the particle and the cone is $\frac{1}{4}$.
The semi-vertical angle of the cone is $30°$ and the particle is a distance ℓ m from the vertex of the cone.

Find the maximum value of ℓ, correct to two places of decimals.





$$R\sin 30 = mg + \mu R\cos 30$$ 5

$$\Rightarrow \quad m = \frac{\left(\frac{1}{2} - \frac{\sqrt{3}}{8}\right)R}{g}$$

$$R\cos 30 + \mu R\sin 30 = mr\omega^2$$ 5

$$R\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{4}R\left(\frac{1}{2}\right) = mr\left(16\pi^2\right)$$

$$\frac{R\sqrt{3}}{2} + \frac{R}{8} = \frac{\left(\frac{1}{2} - \frac{\sqrt{3}}{8}\right)R}{g}r\left(16\pi^2\right)$$ 5

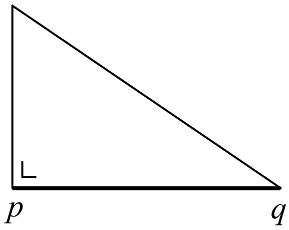
$$\Rightarrow \quad r = 0.2169$$ 5

$$\ell\sin 30 = r$$

$$\ell = 2r$$

$$= 0.43 \text{ m.}$$ 5 | 25

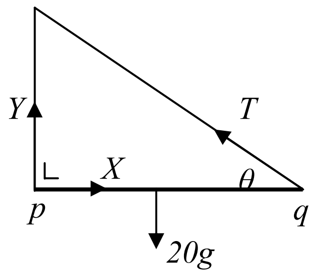
7. **(a)** A uniform rod [*pq*] of length 4 m is free to turn in a vertical plane about a hinge at *p*.
The mass of the rod is 20 kg.
The rod is supported in a horizontal position by a rope attached to *q* and to a point 3 m vertically above *p*.



Find

(i) the tension in the rope

(ii) the magnitude and direction of the reaction at the hinge.



(i) Take moments about p:

$$T\sin\theta(4) = 20g(2)$$ 5

$$T\left(\frac{3}{5}\right)(4) = 20g(2)$$

$$T = \frac{50g}{3} \quad \text{or} \quad 163.3 \text{ N}$$ 5

(ii) $$X = T\cos\theta = \frac{50g}{3}\left(\frac{4}{5}\right) = \frac{40g}{3}$$ 5

$$Y + T\sin\theta = 20g$$

$$Y + 10g = 20g$$

$$Y = 10g$$ 5

magnitude of reaction $= \sqrt{X^2 + Y^2}$

$$= \sqrt{\left(\frac{40g}{3}\right)^2 + (10g)^2}$$

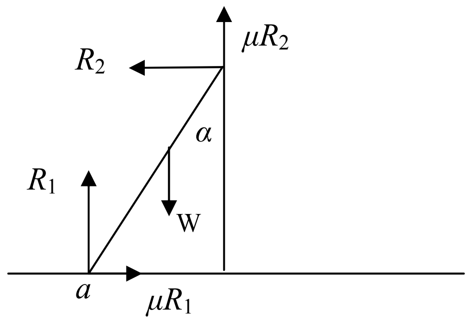
$$= \frac{50g}{3} \quad \text{or} \quad 163.3 \text{ N}$$

direction $\tan\alpha = \frac{Y}{X} = \frac{10g}{\frac{40g}{3}} = \frac{3}{4}$

$$\alpha = 36.87° \quad \text{or} \quad \tan^{-1}\left(\tfrac{3}{4}\right)$$ 5 25

7 **(b)** One end of a uniform ladder rests on a rough horizontal floor and the other end rests against a rough vertical wall.
The coefficient of friction at each contact is μ.
The ladder makes an acute angle α with the wall.

If the ladder is on the point of slipping find α in terms of λ, the angle of friction, where $\mu = \tan\lambda$.



horiz $\quad \mu R_1 = R_2$ **5**

vert $\quad R_1 + \mu R_2 = W$ **5**

$$R_2 = \frac{\mu W}{1+\mu^2}$$

Take moments about a for system:

$$\mu R_2\,(\ell\sin\alpha) + R_2\,(\ell\cos\alpha) = W\left(\tfrac{1}{2}\ell\sin\alpha\right)$$ **5**

$$\mu R_2 \tan\alpha + R_2 = \tfrac{1}{2}W\tan\alpha$$

$$\mu\left(\frac{\mu W}{1+\mu^2}\right)\tan\alpha + \left(\frac{\mu W}{1+\mu^2}\right) = \tfrac{1}{2}W\tan\alpha$$

$$\frac{\mu}{1+\mu^2} = \tan\alpha\left(\frac{1}{2} - \frac{\mu^2}{1+\mu^2}\right)$$

$$\frac{2\mu}{1-\mu^2} = \tan\alpha$$ **5**

$$\frac{2\tan\lambda}{1-\tan^2\lambda} = \tan\alpha$$

$$\tan 2\lambda = \tan\alpha$$

$$\Rightarrow \alpha = 2\lambda$$ **5**

25

8. **(a)** Prove that the moment of inertia of a uniform rod of mass *m* and length 2ℓ about an axis through its centre perpendicular to the rod is $\dfrac{1}{3}m\,\ell^2$.

Let M = mass per unit length

mass of element $= M\{dx\}$

moment of inertia of the element $= M\{dx\}x^2$ 5

moment of inertia of the rod $= M\int_{-\ell}^{\ell} x^2\,dx$ 5

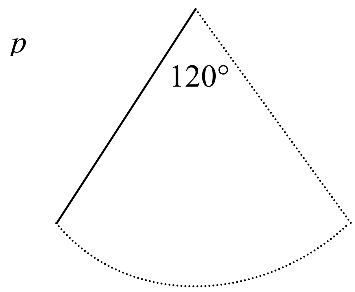
$$= M\left[\frac{x^3}{3}\right]_{-\ell}^{\ell}$$ 5

$$= \tfrac{2}{3}M\,\ell^3$$

$$= \tfrac{1}{3}m\,\ell^2$$ 5 | 20

8 **(b)** A uniform rod of mass $3m$ and length 1.2 metres can turn freely in a vertical plane about a horizontal axis through one end.

The rod oscillates through an angle of $120°$, as shown in the diagram.



(i) Find the angular velocity of the rod when the rod is vertical.

(ii) Find, in terms of m, the vertical thrust on the axis when the rod is vertical.

(i)

$$I = \frac{4}{3}(3m)(0.6)^2 \text{ or } 1.44m$$ 5

$$h = 0.6 - 0.6\cos 60$$ 5

$$\text{Gain in KE} = \text{Loss in PE}$$

$$\tfrac{1}{2} I\omega^2 = mgh$$

$$\frac{1}{2}\left\{\frac{4}{3}(3m)(0.6)^2\right\}\omega^2 = (3m)g(0.6 - 0.6\cos 60)$$ 5

$$\omega^2 = \frac{5g}{4} \quad \text{or} \quad 12.25$$

$$\omega = 3.5 \quad \text{rad/s}$$ 5

(ii)

$$R - 3mg = (3m)r\omega^2$$ 5

$$r = 0.6$$

$$R - 3mg = (3m)(0.6)(12.25)$$

$$R = 51.45m \text{ or } 5.25mg \text{ N}$$ 5

30

9. **(a)** $16\ \text{cm}^3$ of acid of relative density 1.8 is mixed with $7\ \text{cm}^3$ of water to form $22.35\ \text{cm}^3$ of dilute acid.

Find

(i) the contraction in volume which has taken place.

(ii) the relative density of the dilute acid.

(i) Contraction in volume $= 16 + 7 - 22.35$

$$= 0.65\ \text{cm}^3$$ 5

(ii) Mass of dilute acid = Mass of water + Mass of acid

$$(1000\,s)(22.35)10^{-6} =$$ 5

$$1000(7)10^{-6}$$ 5

$$+1800(16)10^{-6}$$ 5

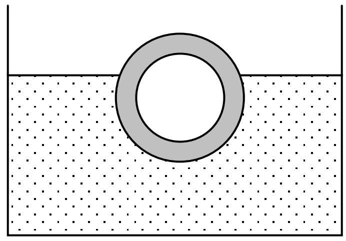
$$22.35s = 35.8$$

$$s = \frac{35.8}{22.35} = 1.6$$ 5

25

9 **b)** A hollow spherical shell of external radius a and internal radius b floats in water.

The relative density of the material of the shell is s.



(i) Find the maximum buoyancy force that the shell could experience while floating in water.

(ii) Hence, or otherwise, prove that if the shell is not totally immersed in the water then

$$s < \frac{a^3}{a^3 - b^3}.$$

(iii) If $s = \dfrac{27}{19}$, prove that the thickness of the shell must be less than $\dfrac{a}{3}$.

(i)

$$\text{Maximum Buoyancy} = \rho V g$$
$$= 1000\left\{\frac{4}{3}\pi a^3\right\}g$$
$$= \frac{4000\pi a^3 g}{3}$$ 5

(ii)

$$B = W$$
$$\Rightarrow W < \text{ Maximum Buoyancy}$$ 5

$$(1000s)\left\{\frac{4}{3}\pi\left(a^3 - b^3\right)\right\}g < \frac{4000\pi a^3 g}{3}$$ 5
$$s\left(a^3 - b^3\right) < a^3$$
$$s < \frac{a^3}{\left(a^3 - b^3\right)}$$ 5

(iii)

$$\frac{27}{19} < \frac{a^3}{\left(a^3 - b^3\right)}$$
$$2a < 3b$$
$$\text{thickness } = a - b < a - \frac{2a}{3}$$
$$\Rightarrow \text{ thickness} < \frac{a}{3}$$ 5

25

10. **(a)** Solve the differential equation

$$\frac{dy}{dx} = \frac{xy}{1+x}$$

given that $y = e$ when $x = 0$.

$$\frac{dy}{dx} = \frac{xy}{1+x}$$

$$\int \frac{dy}{y} = \int \frac{xdx}{1+x} \qquad 5$$

$$= \int \left(1 - \frac{1}{1+x}\right) dx$$

$$\ln y = x - \ln(1+x) + C \qquad 5$$

$$y = e,\ x = 0 \ \Rightarrow\ \mathrm{C} = 1 \qquad 5$$

$$\ln y = x - \ln(1+x) + 1$$

$$\mathrm{y} = e^{x - \ln(1+x) + 1} \text{ or } \frac{e^{x+1}}{1+x} \qquad 5 \qquad 20$$

10 **(b)** The acceleration of a particle moving horizontally in a straight line is $\frac{1}{x^3}$ away from a fixed point o, where x is its distance from o.

The particle starts from rest at $x = 1$.

Calculate

(i) the velocity of the particle when $x = \frac{4}{3}$

(ii) the time that it takes the particle to reach a point 2 metres from o.

$(i)\quad v\frac{dv}{dx} = x^{-3}$	5	
$\int_0^v v\, dv = \int_1^{4/3} x^{-3}\, dx$	5	
$\left[\frac{1}{2}v^2\right]_0^v = \left[-\frac{1}{2x^2}\right]_1^{4/3}$	5	
$\frac{1}{2}v^2 = -\frac{9}{32} + \frac{1}{2}$	5	
$v = \frac{\sqrt{7}}{4}$ or 0.66 m/s	5	
$(ii)\quad \left[\frac{1}{2}v^2\right]_0^v = \left[-\frac{1}{2x^2}\right]_1^x$		
$\frac{1}{2}v^2 = -\frac{1}{2x^2} + \frac{1}{2}$		
$v = \sqrt{1 - \frac{1}{x^2}} = \sqrt{\frac{x^2 - 1}{x^2}}$		
$\int_1^2 \frac{x\,dx}{\sqrt{x^2 - 1}} = \int_0^t dt$		
$\left[\sqrt{x^2 - 1}\right]_1^2 = t$		
$t = \sqrt{3}$ or 1.73 s	5	30

Some Alternative Solutions

1. **(a)**

(ii)

$$\left.\begin{array}{lcl} v = u + at & \text{and} & v = u + at \\ v = 0 + ft_1 & & 0 = v - 3ft_2 \\ \dfrac{v}{f} = t_1 & & \dfrac{v}{3f} = t_2 \end{array}\right\}$$

5

$$t_1 + t_2 = t$$

$$\frac{v}{f} + \frac{v}{3f} = t$$

$$v = \frac{3ft}{4}$$

5

$$d = \tfrac{1}{2}(t_1 + t_2)v \text{ or } \tfrac{1}{2}tv$$

5

$$= \frac{1}{2}t\left(\frac{3ft}{4}\right)$$

$$= \tfrac{3}{8}ft^2$$

5 20

(ii)

$$\left.\begin{array}{lcl} v = u + at & \text{and} & v = u + at \\ v = 0 + ft_1 & & 0 = v - 3ft_2 \\ \dfrac{v}{f} = t_1 & & \dfrac{v}{3f} = t_2 \end{array}\right\}$$

5

$$t_1 + t_2 = t$$

$$\frac{v}{f} + \frac{v}{3f} = t$$

$$v = \frac{3ft}{4}$$

5

$$d = s_1 + \mathrm{s}_2$$

$$= \tfrac{1}{2}ft_1^2 + vt_2 - \tfrac{3}{2}ft_2^2$$

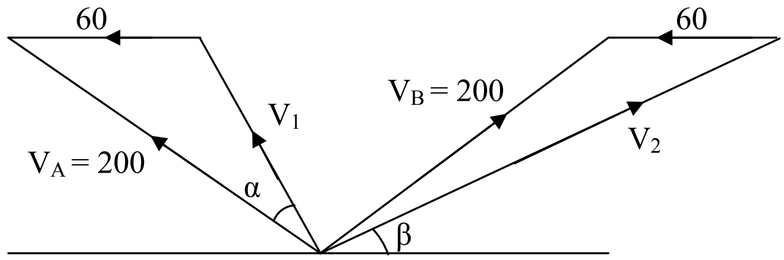
5

$$= \frac{2v^2}{3f} = \frac{2}{3f}\left(\frac{3ft}{4}\right)^2$$

$$= \tfrac{3}{8}ft^2$$

5 20

2 **(a)**



(*i*) $v_1^2 = 60^2 + 200^2 - 2(60)(200)\cos 45$ 5

$= 26629.43725$

$v_1 = 163.185285$ km/h 5

(*ii*) $v_2^2 = 60^2 + 200^2 - 2(60)(200)\cos 135$

$= 60570.56275$

$v_2 = 246.11088$ km/h

$\vec{V}_{12} = \vec{V}_1 - \vec{V}_2$ 5

$= \left(-81.4\vec{i} + 141.4\vec{j}\right) - \left(201.4\vec{i} + 141.4\vec{j}\right)$

$= -282.8\vec{i} + 0\vec{j}$ 5

$$\left.\begin{array}{ll}\text{magnitude:} & 282.84 \text{ km/h} \\ \text{direction:} & \text{west}\end{array}\right\}$$ 5

25

3(a)

(iii)

$$\vec{r} = (gt)\ \vec{i} + \left(3gt - \tfrac{1}{2}gt^2\right)\vec{j}$$
$$\vec{v} = (g)\ \vec{i} + (3g - gt)\ \vec{j}$$

$\vec{r}$ and $\vec{v}$ are perpendicular

$$\left\{(gt)\ \vec{i} + \left(3gt - \tfrac{1}{2}gt^2\right)\vec{j}\right\}\left\{(g)\ \vec{i} + (3g - gt)\ \vec{j}\right\} = 0 \qquad 5$$

$$gt(g) + \left(3gt - \tfrac{1}{2}gt^2\right)(3g - gt) = 0$$

$$1 + 9 - \tfrac{3}{2}t - 3t + \tfrac{1}{2}t^2 = 0$$

$$t^2 - 9t + 20 = 0$$

$$(t-4)(t-5) = 0$$

$$\Rightarrow \quad t = 4\text{ s} \quad \text{and} \quad t = 5\text{ s} \qquad 5 \qquad \boxed{10}$$