2000 HIGHER LEVEL SOLUTION & MARKING SCHEME

**1.** **(a)** A stone projected vertically upwards with an initial speed of $u$ m/s rises 70 m in the first $t$ seconds and another 50 m in the next $t$ seconds.

Find the value of $u$.

$$s = ut + \tfrac{1}{2}ft^2$$

$$70 = ut - \tfrac{1}{2}gt^2$$ **5**

$$120 = u(2t) - \tfrac{1}{2}g(2t)^2 \quad or \quad 50 = (u - gt)t - \tfrac{1}{2}gt^2$$ **5**

$$140 = 2ut - gt^2$$

$$120 = 2ut - 2gt^2$$

$$20 = gt^2$$

$$\Rightarrow t = \sqrt{\frac{20}{g}}$$ **5**

$$140 = 2u\left(\sqrt{\frac{20}{g}}\right) - 20$$

$$\Rightarrow u = 56 \text{ ms}^{-1}$$ **5** **20**

2000

1\. **(b)** A car, starting from rest and travelling from $p$ to $q$ on a straight level road, where $|pq| = 10\,000$ m, reaches its maximum speed 25 m/s by constant acceleration in the first 500 m and continues at this maximum speed for the rest of the journey.

A second car, starting from rest and travelling from $q$ to $p$, reaches the same maximum speed by constant acceleration in the first 250 m and continues at this maximum speed for the rest of the journey.

**(i)** If the two cars start at the same time, after how many seconds do the two cars meet ? Find, also, the distance travelled by each car in that time.

**(ii)** If the start of one car is delayed so that they meet each other exactly halfway between p and q, find which car is delayed and by how many seconds.

$$\tfrac{1}{2}t_p(25) = 500 \Rightarrow t_p = 40 \text{ and}$$
$$\tfrac{1}{2}t_q(25) = 250 \Rightarrow t_q = 20$$

Time to reach maximum speed: 40 s for p and 20 s for q — 5

$s_p = 500 + 25(t-40)$ and $s_q = 250 + 25(t-20)$ — 5

$s_p + s_q = 10000 \Rightarrow t = 215$ — 5

$s_p = 500 + 25(215-40) = 4875$ and
$s_q = 250 + 25(215-20) = 5125$ — 5

$s_q > s_p \Rightarrow$ q is delayed by $t_1$ seconds — 5

$s_p = s_q \Rightarrow 500 + 25(t-40) = 250 + 25(t - t_1 - 20)$
$\Rightarrow t_1 = 10$ seconds — 5 — **30**

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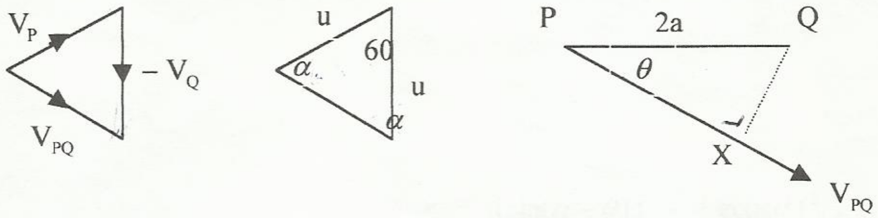
**2 cont.**




$$\vec{V}_{PQ} = \vec{V}_P - \vec{V}_Q$$

$$\alpha = 60° \quad \Rightarrow \quad \theta = 30° \qquad 5$$

$$\text{shortest distance} = 2a\sin\theta = a \qquad 5$$

$$\text{time to reach X} = t = \frac{2a\cos\theta}{V_{PQ}} = \frac{a\sqrt{3}}{u} \qquad 5$$

$$\text{Distance moved by P} = 2u\left(\frac{T}{2}\right) + ut = \frac{4a}{\sqrt{3}} + a\sqrt{3} = \frac{7a}{\sqrt{3}} \qquad 5$$

$$\text{Distance moved by Q} = u\left(\frac{T}{2}\right) + ut = \frac{2a}{\sqrt{3}} + a\sqrt{3} = \frac{5a}{\sqrt{3}} \qquad 5 \qquad 50$$

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**2.** At a certain instant ship Q is at a distance of $4a$ due east of ship P. Q is moving northwards with constant speed $u$ and P is travelling with constant speed $2u$.

Find the direction of P if it is to intercept Q.
Find the time $T$, in terms of $a$ and $u$, it would take P to intercept Q.

If, instead, after time $\frac{T}{2}$ has elapsed, the speed of P drops to constant speed $u$ without changing direction, find, in terms of $a$,

**(i)** the shortest distance between P and Q

**(ii)** the distance each ship has moved from its original position to its position when they are closest together.

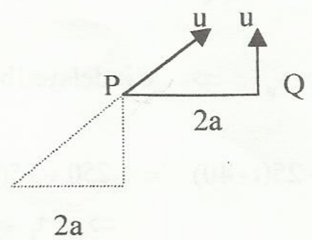
Direction : $2u\sin\alpha = u$

$\Rightarrow \sin\alpha = \frac{1}{2}$ or $\alpha = 30°$ 10

Time : $2u\cos\alpha\,(T) = 4a \Rightarrow T = \dfrac{4a}{u\sqrt{3}}$ 10

After $\frac{T}{2}$ seconds P has moved $2u\cos\alpha\left(\frac{T}{2}\right) = 2a$ East 5

New position of P and Q



3. (a) A particle is projected with a velocity of $u$ m/s at an angle β to the horizontal ground.

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Show that the particle hits the ground at a distance $\frac{u^2}{g}\sin 2\beta$ from the point of projection.

Find the angle of projection which gives maximum range.

$$\vec{r} = (u\cos\beta . t)\,\vec{i} + (u\sin\beta . t - \tfrac{1}{2}gt^2)\,\vec{j}$$ 5

$$r_j = 0 \quad \Rightarrow \quad t = \frac{2u\sin\beta}{g}$$ 5

$$\text{Range} = u\cos\beta\left(\frac{2u\sin\beta}{g}\right) = \frac{u^2\sin 2\beta}{g}$$ 5

$$\text{For max range} \quad \sin 2\beta = 1 \quad \text{or} \quad \frac{dR}{d\beta} = \frac{u^2\cos 2\beta(2)}{g} = 0$$

$$\Rightarrow \quad \beta = 45^\circ$$ 5 20

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**(b)** A particle is projected at an angle $\alpha = \tan^{-1} 3$ **to the horizontal** up a plane inclined at an angle θ to the horizontal. (The plane of projection is vertical and contains the line of greatest slope). The particle strikes the plane at right angles.

Find two possible values for θ.

$$r_j = u\sin(\alpha - \theta).t - \tfrac{1}{2} g\cos\theta.t^2$$ 5

$$v_i = u\cos(\alpha - \theta) - g\sin\theta.t$$ 5

$$r_j = 0 \Rightarrow t = \frac{2u\sin(\alpha - \theta)}{g\cos\theta}$$ 5

$$v_i = 0 \Rightarrow t = \frac{u\cos(\alpha - \theta)}{g\sin\theta}$$ 5

$$\frac{2u\sin(\alpha - \theta)}{g\cos\theta} = \frac{u\cos(\alpha - \theta)}{g\sin\theta}$$

$$2\tan(\alpha - \theta).\tan\theta = 1$$

$$2\left\{\frac{\tan\alpha - \tan\theta}{1 + \tan\alpha.\tan\theta}\right\}\tan\theta = 1 \Rightarrow 2\left\{\frac{3 - \tan\theta}{1 + 3\tan\theta}\right\}\tan\theta = 1$$ 5

$$2\tan^2\theta - 3\tan\theta + 1 = 0$$

$$\Rightarrow \tan\theta = \tfrac{1}{2} \text{ or } \tan\theta = 1$$ 5

$$\Rightarrow \theta = 26.6° \text{ or } 45°$$ 30

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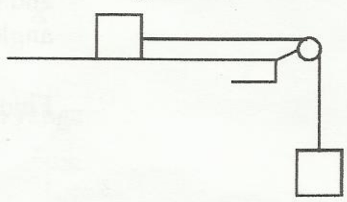
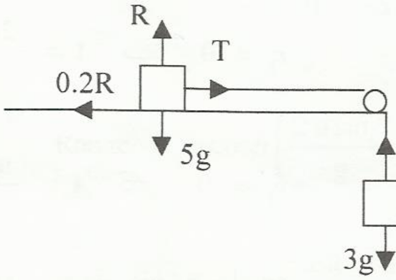
**4.** **(a)** A mass of 5 kg on a rough horizontal table is connected by a light inextensible string passing over a smooth light pulley, at the edge of the table, to a 3 kg mass hanging freely. The coefficient of friction between the 5 kg mass and the table is $\frac{1}{5}$.

The system is released from rest.

Find the distance fallen by the 3 kg mass in the first 2 seconds after the system is released from rest.




$$T - g = 5a$$ 5

$$3g - T = 3a$$ 5

$$\Rightarrow \quad a = \tfrac{1}{4}g$$ 5

$$s = ut + \tfrac{1}{2}at^2$$

$$= 0 + \tfrac{1}{2}(\tfrac{1}{4}g)(4)$$

$$= \tfrac{1}{2}g \quad or \quad 4.9$$ 5

20

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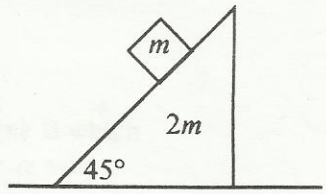
4. **(b)** A smooth wedge of mass $2m$ and slope 45° is placed on a smooth horizontal surface. A particle of mass $m$ is placed on the inclined face of the wedge.
The system is released from rest.

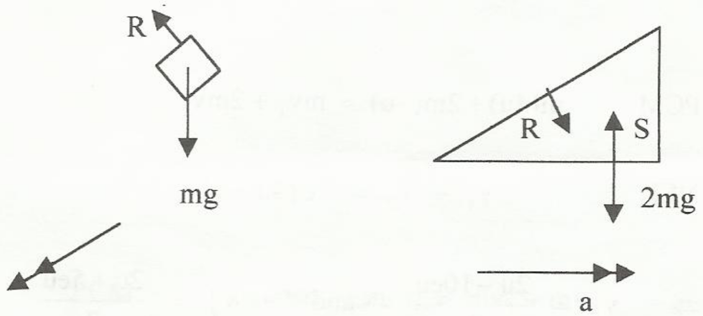



**(i)** Show on separate diagrams the forces acting on the wedge and the particle.

**(ii)** Show that the acceleration of the wedge is $\frac{g}{5}$ m/s$^2$.

**(iii)** Find the speed of the particle relative to the wedge, when the speed of the wedge is 1 m/s.

(i)

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(ii)

$$R\sin 45 = 2ma \quad (3)$$

$$mg\cos 45 - R = ma\sin 45 \quad (1)$$

$$\frac{mg}{\sqrt{2}} - 2\sqrt{2}\,ma = m\frac{a}{\sqrt{2}}$$

$$\Rightarrow \quad a = \tfrac{1}{5}g \quad (4)$$

5
5
5

(iii)

$$mg\sin 45 = m(b - a\cos 45) \quad (2)$$

$$\frac{mg}{\sqrt{2}} = m\left(b - \frac{g}{5\sqrt{2}}\right)$$

$$\Rightarrow \quad b = \frac{6g}{5\sqrt{2}}$$

5

Wedge: $v = u + at$

$$1 = 0 + \tfrac{1}{5}gt \quad \Rightarrow \quad t = \frac{5}{g}$$

Particle: $v = u + bt$

$$v = 0 + \frac{6g}{5\sqrt{2}}\left(\frac{5}{g}\right) = \frac{6}{\sqrt{2}} \text{ or } 3\sqrt{2}$$

5

30

5. (a) Two smooth spheres whose masses are $m$ and $2m$ move towards each other in a straight line with speeds $4u$ and $u$, respectively.

Show that the spheres will move in opposite directions after the collision if $e > \frac{1}{5}$, where $e$ is the coefficient of restitution.

2000 (a)

PCM $\quad m(4u) + 2m(-u) = mv_1 + 2mv_2$ 5

NEL $\quad v_1 - v_2 = -e(4u + u)$ 5

$\Rightarrow \quad v_1 = \dfrac{2u - 10eu}{3} \quad \text{and} \quad v_2 = \dfrac{2u + 5eu}{3}$ 5

$v_2 > 0 \quad \forall e \quad \text{as} \quad 0 \le e \le 1$

$v_1 < 0 \quad \text{if} \quad 2u - 10eu < 0$

$1 < 5e \quad \Rightarrow \quad e > \frac{1}{5}$ 5

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2000 **(b)** A smooth sphere A collides with an identical smooth sphere B which is at rest. The velocity of A before impact makes an angle $\alpha$ with the line of centres at impact, where $0° \leq \alpha < 90°$.

The coefficient of restitution between the spheres is $\frac{1}{2}$.

Show that the angle θ through which the path of A is deflected is given by

$$\tan\theta = \frac{3\tan\alpha}{1 + 4\tan^2\alpha}.$$

| | | |
|---|---|---|
| PCM $\quad m(u\cos\alpha) + m(0) = mv_1 + mv_2$ | 5 | |
| NEL $\quad v_1 - v_2 = -\frac{1}{2}(u\cos\alpha - 0)$ | 5 | |
| $\Rightarrow \quad v_1 = \frac{u\cos\alpha}{4}$ | 5 | |
| $\tan\beta = \frac{u\sin\alpha}{\frac{1}{4}u\cos\alpha} = 4\tan\alpha$ | 5 | |
| $\tan\theta = \tan(\beta - \alpha) = \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$ | 5 | |
| $= \frac{4\tan\alpha - \tan\alpha}{1 + 4\tan\alpha\tan\alpha}$ | | |
| $= \frac{3\tan\alpha}{1 + 4\tan^2\alpha}$ | 5 | 30 |

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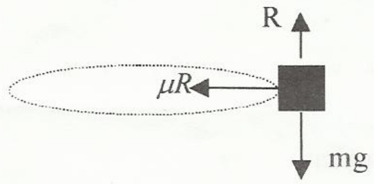
**6.** **(a)** A particle is placed on a horizontal rotating turntable, 10 cm from the centre of rotation. There is a coefficient of friction of 0.4 between the particle and the turntable. If the speed of the turntable is gradually increased, at what angular speed will the particle begin to slide?




$$R = mg$$ 5

$$\mu R = m\,r\,\omega^2$$ 5

$$0.4(mg) = m\,(0.1)\,\omega^2$$ 5

$$\Rightarrow \quad \omega = \sqrt{4g} \quad \text{or} \quad 6.26 \text{ rad s}^{-1}$$ 5

$\therefore$ The particle slides if $\omega > \sqrt{4g}$

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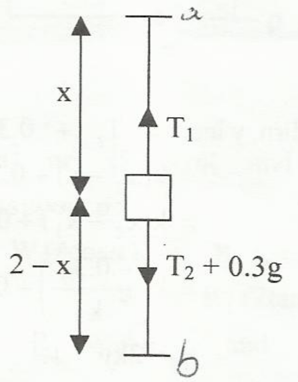
**(b)** A particle of mass 0.3 kg is attached to the midpoint of a light elastic string of natural length 1 m and elastic constant $k$. The string is then stretched between two points $a$ and $b$. The point $a$ is 2 m vertically above $b$.

Find

**(i)** the extensions of the two parts of the string, in terms of $k$, when the system is in equilibrium

**(ii)** the minimum value of $k$ which will ensure that the lower part of the string is taut

**(iii)** the period of small oscillations, in terms of $k$, when the particle is displaced vertically. (Assume both parts of the string remain taut.)




(i)

$$T_1 = T_2 + 0.3g$$

$$k(x - \tfrac{1}{2}) = k(1\tfrac{1}{2} - x) + 0.3g$$

$$\Rightarrow \quad x = 1 + \frac{0.15g}{k}$$

$$x_1 = x - \tfrac{1}{2} = \frac{1}{2} + \frac{0.15g}{k} \quad \text{or} \quad \frac{k + 0.3g}{2k}$$ 5

$$x_2 = 1\tfrac{1}{2} - x = \frac{1}{2} - \frac{0.15g}{k} \quad \text{or} \quad \frac{k - 0.3g}{2k}$$ 5

(ii) $x_2 \geq 0 \quad \Rightarrow \quad k \geq 0.3g$

$\Rightarrow$ min value of $k = 0.3g$ 5

5 20

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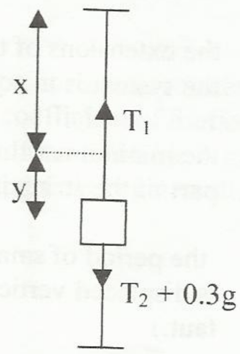
**6(b) cont.**




(iii) Force in dirn. y inc. $= T_2 + 0.3g - T_1$

$$= k(x_2 - y) + 0.3g - k(x_1 + y)$$

$$= k(x_2 - x_1) + 0.3g - 2ky$$

$$= k\left(\frac{-0.3g}{k}\right) + 0.3g - 2ky$$

$$= -2ky$$

$\Rightarrow$ acceleration $= \dfrac{-2k}{0.3}y$

$\therefore$ SHM about $y = 0$ with $\omega = \sqrt{\dfrac{2k}{0.3}}$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{0.3}{2k}} \text{ or } \pi\sqrt{\frac{0.6}{k}}$$

5

5

10

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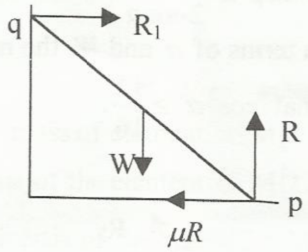
7. (a) A uniform ladder [ $pq$], of length $2l$ and weight $W$, is in equilibrium with end $p$ against a rough horizontal floor and end $q$ against a smooth vertical wall. The ladder makes an angle $\tan^{-1} 2$ with the floor.

**(i)** Show that the least possible value for $\mu$, the coefficient of friction between the ladder and the floor, is $\frac{1}{4}$.

**(ii)** If, however, $\mu = \frac{1}{3}$, find, in terms of $l$, the distance from $p$ of the highest point on the ladder at which a man of weight $2W$ can stand without the ladder slipping.




(i)

$\{\text{diagram}\}$ or $\{R_1 = \mu R \text{ and } R = W\}$ — 5

Moments about p:

$$W(\ell\cos\alpha) = R_1(2\ell\sin\alpha)$$ — 5

$$W = R_1(2\tan\alpha)$$

$$\Rightarrow \quad R_1 = \tfrac{1}{4}W \quad \text{and} \quad \mu = \tfrac{1}{4}$$ — 5

(ii)

$$R_1 = \mu R \quad (= \tfrac{1}{3}R = W)$$

$$R = 3W$$

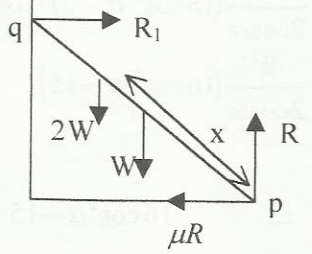
Moments about p:

$$W(\ell\cos\alpha) + 2W(x\cos\alpha) = R_1(2\ell\sin\alpha)$$ — 5

$$W\ell + 2Wx = R_1(2\ell\tan\alpha)$$

$$= 4W\ell$$

$$\Rightarrow \quad x = \frac{3\ell}{2}$$ — 5 — 25



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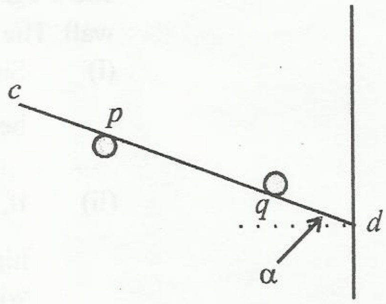
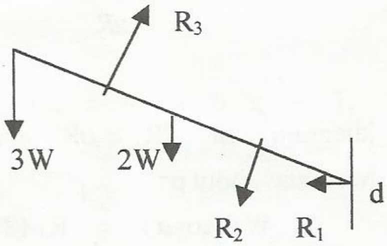
**(b)** A uniform rod $[cd]$ of weight $2W$ rests in equilibrium at an angle $\alpha$ to the horizontal with end $d$ in contact with a smooth vertical wall. The rod passes over a smooth fixed peg at $p$ and under a smooth fixed peg at $q$.
A weight of $3W$ is suspended from $c$.
$|cp| = |qd| = l$ and $|pq| = 2l$.




**(i)** Show that the magnitude of the reaction at $p$ is $\frac{W}{2\cos\alpha}(16\cos^2\alpha - 5)$.

**(ii)** Find, in terms of $\alpha$ and $W$, the magnitude of the reaction at $q$.

**(iii)** Show that $\cos^2\alpha \geq \frac{15}{16}$.




(i)

vert: $R_3\cos\alpha = R_2\cos\alpha + 5W$ — 5

Moments about d:

$R_2(\ell) + 2W(2\ell\cos\alpha) + 3W(4\ell\cos\alpha) = R_3(3\ell)$ — 5

$\left(R_3 - \frac{5W}{\cos\alpha}\right) + 16W\cos\alpha = 3R_3$

$\Rightarrow \quad R_3 = \frac{W}{2\cos\alpha}\left\{16\cos^2\alpha - 5\right\}$ — 5

(ii)

$$R_2 = 3R_3 - 16W\cos\alpha$$

$$= \frac{3W}{2\cos\alpha}(16\cos^2\alpha - 5) - 16W\cos\alpha$$

$$= \frac{W}{2\cos\alpha}(16\cos^2\alpha - 15)$$ — 5

(iii)

$R_2 \geq 0 \quad \Rightarrow \quad 16\cos^2\alpha - 15 \geq 0$

$\cos^2\alpha \geq \frac{15}{16}$ — 5

25

**8.** **(a)** Prove that the moment of inertia of a uniform disc, of mass *m* and radius *r*, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

Let $M$ = mass per unit area

mass of element $= M\{2\pi x.dx\}$ — 5

moment of inertia of the element $= M\{2\pi x.dx\}x^2$ — 5

moment of inertia of the disc $= 2\pi M \int_0^r x^3\,dx$ — 5

$$= 2\pi M\left[\frac{x^4}{4}\right]_0^r$$

$$= \tfrac{1}{2}\pi M r^4$$

$$= \tfrac{1}{2} m r^2$$ — 5

20

**(b)** A uniform disc of mass *m* and radius *r* rolls from rest, without sliding, 30 m down a plane inclined at an angle of 30° to the horizontal.

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**(i)** Find the linear speed of the disc after rolling 30 m down the plane.

**(ii)** Find the time taken for the disc to roll 30 m down the plane, correct to two places of decimals.

**(iii)** The disc is now replaced by a hoop of mass *m* and radius *r*. The hoop rolls from rest, without sliding, 30 m down the plane. Show that the ratio of the acceleration down the plane of the hoop to that of the disc is $\frac{3}{4}$.

(i)

$$\text{Gain in KE} = \text{Loss in PE}$$ 5

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgh$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left\{\tfrac{1}{2}mr^2\right\}\omega^2 = mg\{30\sin 30\}$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{4}mv^2 = 15mg$$

$$v^2 = 20g$$

$$v = \sqrt{20g} \text{ or } 14\ \text{ms}^{-1}$$ 5

(ii)

$$v^2 = u^2 + 2as$$

$$196 = 0 + 2a(30)$$

$$a = \frac{196}{60} \text{ or } \frac{g}{3}$$ 5

$$v = u + at$$

$$14 = 0 + \frac{196}{60}t$$

$$t = \frac{840}{196} = 4.29 \text{ seconds}$$ 5

(iii)

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\{mr^2\}\omega^2 = mg\{30\sin 30\}$$

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = 15mg$$

$$v^2 = 15g$$ 5

$$v^2 = u^2 + 2as$$

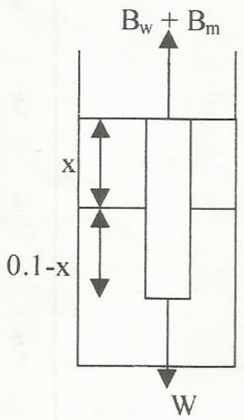
$$15g = 0 + 2a(30)$$

$$a = \frac{147}{60} \text{ or } \frac{g}{4}$$

$$\frac{\text{acc. of hoop}}{\text{acc. of disc}} = \frac{\frac{1}{4}g}{\frac{1}{3}g} = \frac{3}{4}$$ 5 30

**9.** **(a)** A uniform cylinder of height 10 cm floats vertically with half of its height immersed in a container of mercury. The relative density of mercury is 13.6. Water is then poured on top of the mercury until the cylinder is covered.

How far does the cylinder rise?




$$B = W \quad 5$$

$$\frac{\frac{1}{2}W(13.6)}{s} = W$$

$$s = 6.8 \quad 5$$

$$B_w + B_m = W$$

$$\frac{\frac{xW}{0.1}(1)}{6.8} + \frac{\frac{(0.1-x)W}{0.1}(13.6)}{6.8} = W \quad 5$$

$$\Rightarrow \quad x = 0.054 \quad 5$$

Cylinder rises 0.004 m or 0.4 cm. 5

**25**

**OR**

$$B = W \quad 5$$

$$13600(\tfrac{1}{2}V)g = \rho V g$$

$$\rho = 6800 \quad 5$$

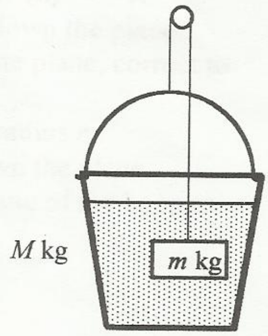
$$B_w + B_m = W \quad 5$$

$$1000(Ax)g + 13600A(0.1-x)g = 6800A(0.1)g$$

$$\Rightarrow \quad x = 0.054 \quad 5$$

Cylinder rises 0.004 m or 0.4 cm. 5

**(b)** A bucket is partially filled with water. The total mass of the bucket and water is $M$ kg. The bucket is attached to one end of a light inextensible string which passes over a smooth light fixed pulley. A block of mass $m$ kg is attached to the other end of the string and is fully immersed in the water in the bucket. The system is in equilibrium and the block does not touch the bucket.




Prove that the relative density of the block is greater than 2.

Block: $T + B = mg$ 5

The block exerts an equal and opposite force on the water 5

Bucket

$$T = B + W$$ 5

$$(mg - B) = B + W$$

$$mg = 2B + W$$

$$mg = 2\frac{mg(1)}{s} + W$$ 5

$$mgs = 2mg + sW$$

$$s = 2 + \frac{sW}{mg} \quad \text{or} \quad s = \frac{2mg}{mg - W}$$

$$\Rightarrow \quad s > 2$$ 5

25

$$2B = mg - W$$

$$2(1000Vg) = 1000sVg - W$$

$$s = 2 + \frac{W}{1000Vg}$$

$$\Rightarrow \quad s > 2$$

**OR**

Block $T = B + mg$ 5

System $2T = mg + W$ 10

$$2(mg - B) = mg + W$$

$$2mg - 2\frac{mg(1)}{s} = mg + W$$ 5

$$s = \frac{2mg}{mg - W}$$

$$\Rightarrow \quad s > 2$$ 5

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**10.** **(a)** If

$$x\frac{dy}{dx}+xy\frac{dy}{dx}-1=0$$

and $y = 2$ when $x = e$, find, correct to two places of decimals, the positive value of $y$ when $x = e^2$.

$$x\frac{dy}{dx}(1+y)=1$$

$$\int_2^y (1+y)dy = \int_e^{e^2} \frac{dx}{x}$$

$$\left[y+\tfrac{1}{2}y^2\right]_2^y = [\ln x]_e^{e^2}$$

$$y+\tfrac{1}{2}y^2-2-2=2-1$$

$$y^2+2y-10=0$$

$$y=2.32$$

| | |
|---|---|
| 5 | |
| 5 | |
| 5 | |
| 5 | 20 |

2000 **(b)** The deceleration of a particle moving in a straight line with speed $v$ m/s has magnitude $4e^{\frac{v}{6}}$ m/s$^2$. The particle has an initial speed of 6 m/s.

**(i)** Find the time $t_1$ for the speed to decrease to 3 m/s.

**(ii)** Find the time $t_2$ for the particle to come to rest.

**(iii)** Deduce that $\frac{t_2 - t_1}{t_1} = \sqrt{e}$.

(i)

$$\frac{dv}{dt} = -4e^{\frac{1}{6}V}$$ 5

$$\int_6^3 e^{-\frac{1}{6}V}\,dv = -4\int_0^{t_1} dt$$ 5

$$-6\left[e^{-\frac{1}{6}V}\right]_6^3 = -4t_1$$ 5

$$t_1 = \frac{6}{4}\left(\frac{1}{\sqrt{e}} - \frac{1}{e}\right)$$ 5

(ii)

$$-6\left[e^{-\frac{1}{6}V}\right]_6^0 = -4t_2$$

$$t_2 = \frac{6}{4}\left(1 - \frac{1}{e}\right)$$ 5

(iii)

$$\frac{t_2 - t_1}{t_1} = \frac{\frac{6}{4}\left\{\left(1-\frac{1}{e}\right) - \left(\frac{1}{\sqrt{e}} - \frac{1}{e}\right)\right\}}{\frac{6}{4}\left(\frac{1}{\sqrt{e}} - \frac{1}{e}\right)}$$

$$= \frac{1 - \frac{1}{\sqrt{e}}}{\frac{1}{\sqrt{e}} - \frac{1}{e}}$$

$$= \frac{e - \sqrt{e}}{\sqrt{e} - 1} = \frac{\sqrt{e}(\sqrt{e} - 1)}{(\sqrt{e} - 1)}$$

$$= \sqrt{e}$$ 5

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