2604

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LEAVING CERTIFICATE EXAMINATION, 1998

APPLIED MATHEMATICS — HIGHER LEVEL

FRIDAY, 26 JUNE – MORNING, 9.30 – 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2.

Marks may be lost if necessary work is not shown.

1. **(a)** A train accelerates uniformly from rest to a speed v m/s. It continues at this constant speed for a period of time and then decelerates uniformly to rest. If the average speed for the whole journey is $\frac{5v}{6}$, find what fraction of the whole distance is described at constant speed.

(b) Car A, moving with uniform acceleration $\frac{3b}{20}$ m/s^2 passes a point p with speed $9u$ m/s. Three seconds later car B, moving with uniform acceleration $\frac{2b}{9}$ m/s^2 passes the same point with speed $5u$ m/s. B overtakes A when their speeds are 6.5 m/s and 5.4 m/s respectively. Find

(i) the value u and the value b

(ii) the distance travelled from p until overtaking occurs.

2. **(a)** The driver of a speedboat travelling in a straight line at 20 m/s wishes to intercept a yacht travelling at 5 m/s in a direction 40° East of North. Initially the speedboat is positioned 5 km South-East of the yacht. Find

(i) the direction of the speedboat if it intercepts the yacht

(ii) how long the journey takes.

(b) A man wishes to row a boat across a river to reach a point on the opposite bank that is 25 m downstream from his starting point. The man can row the boat at 3.2 m/s in still water. The river is 45 m wide and flows uniformly at 3.6 m/s. Find

(i) the two possible directions in which the man could steer the boat

(ii) the respective crossing times.

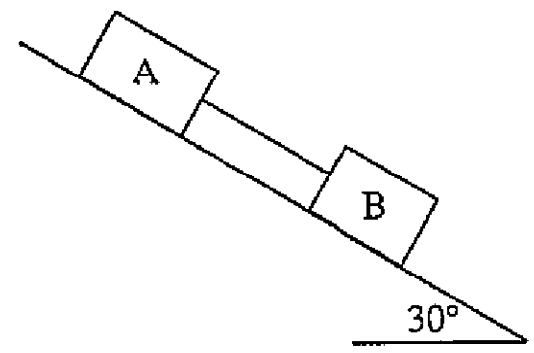
3. **(a)** A football is kicked from a spot on level ground with a velocity of $\sqrt{8g}$ m/s and strikes a vertical wall 4 m away at a point 2 m above the ground. Find the two possible angles of projection.

(b) A particle is projected down a slope which is inclined at 45° to the horizontal. The particle is projected from a point on the slope and has an initial velocity of $7\sqrt{2}$ m/s at an angle α to the inclined plane. Find the value of α if

(i) the particle first hits the slope after 2 seconds

(ii) the landing angle with the slope is $\tan^{-1}\left(\frac{1}{3}\right)$.

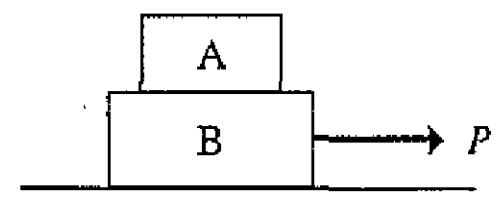
4. **(a)** Blocks A and B, of mass 15 kg and 25 kg, respectively, are connected by a light, inextensible string as shown in the diagram. The coefficients of friction are 0.4 for block A and 0.2 for block B. The blocks move down the plane which is inclined at 30° to the horizontal. Find



(i) the acceleration of block B

(ii) the tension in the string.

(b) The two blocks shown in the diagram are at rest on a horizontal surface when a force P is applied to block B. Blocks A and B have masses 20 kg and 35 kg, respectively. The coefficient of friction between the two blocks is 0.35 and the coefficient of friction between the horizontal surface and block B is 0.3.



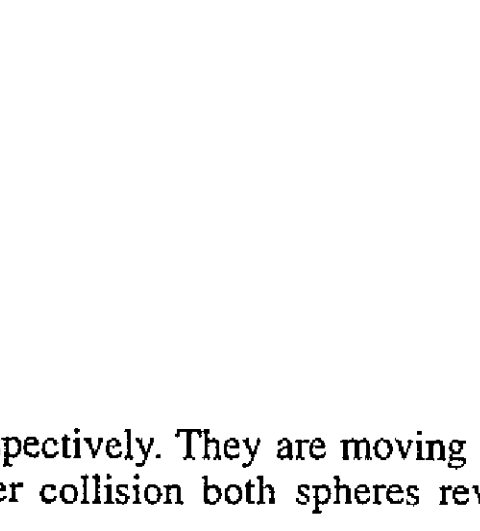
Determine the maximum force, P, before A slips on B.

5. **(a)** Two smooth spheres A and B have masses m_1 and m_2, respectively. They are moving towards each other along the same horizontal line each with speed $2u$. After collision both spheres reverse their original directions of motion and A now travels with speed u.

(i) Show that $3m_1 > 2m_2$.

(ii) Find an expression for e, the coefficient of restitution, and hence or otherwise show that $3m_1 \leq 5m_2$.

(b) A sphere of mass 4 kg is released from rest when $\alpha = 60°$. It swings down and strikes a 7 kg box B when the string is vertical. The distance from the point of support to the centre of the sphere is one metre and the coefficient of restitution for the collision is $\frac{3}{4}$.



Calculate the speed of the box immediately after the impact if the box is free to move.

6. **(a)** Define Simple Harmonic Motion.

The distance, x, of a particle from a fixed point, o, is given by $x = 7 \sin \omega t + 24 \cos \omega t$, ω being a constant.

(i) Show that the particle is describing simple harmonic motion about o.

(ii) Calculate the amplitude of the motion.

(b) An elastic string of natural length one metre is extended 20 cm by a particle attached to its end and hanging freely. The particle is then pulled down a further distance of 40 cm and released.

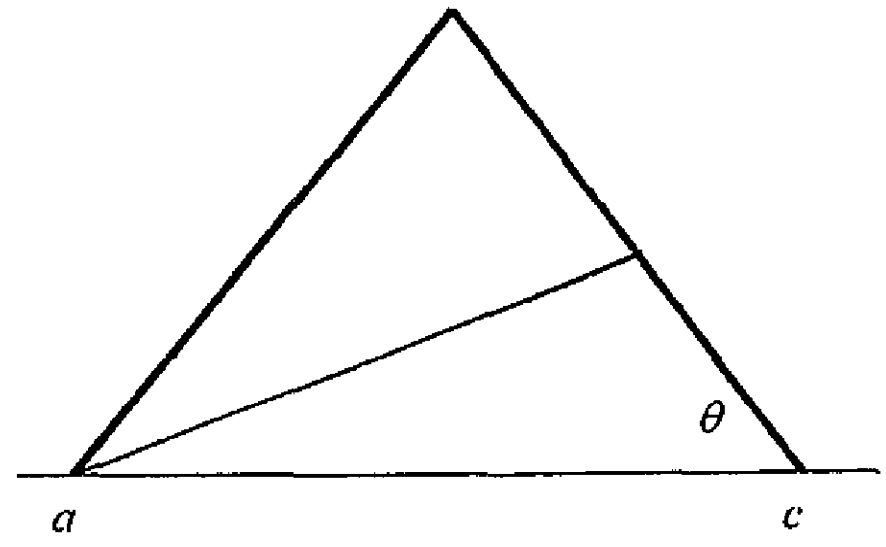
(i) Show that the particle moves with simple harmonic motion when the string is taut.

(ii) Find the height above the equilibrium position to which the particle will rise.

7. Two equal uniform rods $[ab]$ and $[bc]$, each of weight W, are freely jointed at b. An inextensible string connects a to the midpoint of $[bc]$. When the string is taut the angle bca is θ. The rods are placed in a vertical plane with a and c on a smooth horizontal surface.

Prove that the tension in the string is

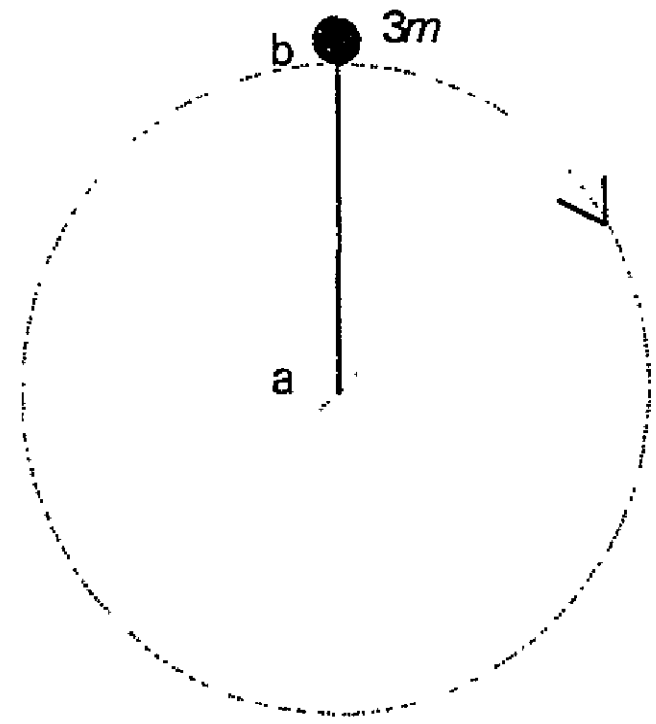
$$\frac{W}{4}\sqrt{1 + 9\cot^2\theta}.$$



8. **(a)** Prove that the moment of inertia of a uniform rod $[ab]$ of mass m and length $2l$ about an axis through a, perpendicular to the rod, is $\frac{4}{3}ml^2$.

(b) A lamina is rotating with angular velocity ω about an axis perpendicular to its plane. If the moment of inertia of the lamina about the axis is I, prove that the kinetic energy is $\frac{1}{2}I\omega^2$.

(c) A uniform rod $[ab]$, of mass m and length $2l$, is free to rotate in a vertical plane about a fixed horizontal axis at a, with a particle of mass $3m$ attached to the rod at b. The system is released from rest with the rod vertical and the end b above a.



(i) Show that the angular velocity of the rod when next it is vertical is $\sqrt{\frac{21g}{10l}}$.

(ii) If at this point the mass falls off, find the height to which the end b subsequently rises.

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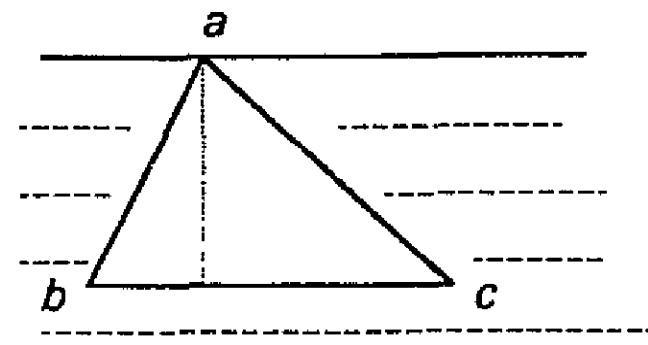
9. **(a)** A triangular lamina abc is immersed in a vertical position in water with its vertix a at the surface and its base $[bc]$ parallel to the surface.



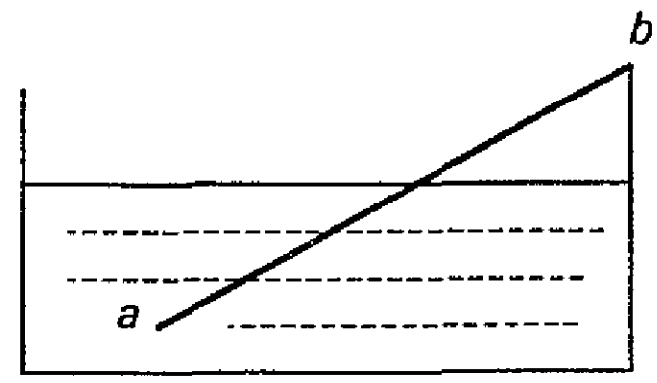
(i) If $|bc| = 10$ cm and the height of the triangle is 7.5 cm, find the thrust on abc due to the water.

(ii) If d and f are the midpoints of $[ab]$, $[ac]$

respectively, find the ratio

$$\frac{\text{thrust on } adf}{\text{thrust on } dbcf}$$

(b) A thin uniform rod $[ab]$ of length l and relative density s is in equilibrium in an inclined position with the end a immersed in a container of water and the end b supported on the edge of the container.



Show that the length of the immersed part of the rod is

$$l\,(1 - \sqrt{1 - s}).$$

10. **(a)** If

$$t\frac{dv}{dt} = v - vt$$

and $v = 3$ when $t = 5$,

find the value of $\ddot{v}$ when $t = 6$.

(b) A particle moves in a straight line. The initial speed is u and the retardation is kv^3, where v is the speed at the time t. If s is the distance travelled in time t, prove

(i) $v = \dfrac{u}{1 + ksu}$

(ii) $t = \dfrac{ks^2}{2} + \dfrac{s}{u}$.