AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1996

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 21 JUNE – MORNING, 9.30 to 12.00

2440

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2 .

Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

1. *(a)* A particle starts from rest and moves in a straight line with uniform acceleration. It passes three points a, b and c where $|ab| = 105$ m and $|bc| = 63$ m. If it takes 6 seconds to travel from a to b and 2 seconds to travel from b to c find

(i) its acceleration

(ii) the distance of a from the starting position.

(b) A lift starts from rest with constant acceleration 4 m/s^2. It then travels with uniform speed and finally comes to rest with constant retardation 4 m/s^2. The total distance travelled is d and the total time taken is t.

(i) Draw a speed-time graph.

(ii) Show that the time for which it travelled with uniform speed is

$$\sqrt{t^2 - d}.$$

2. A ship, B, is travelling due West at 25.6 km/h. A second ship, C, travelling at 32 km/h is first sighted 17 km due North of B. From B the ship C appears to be moving South-east.

Find

(i) the direction in which C is actually moving

(ii) the velocity of C relative to B

(iii) the shortest distance between the ships in the subsequent motion

(iv) the time that elapses, after first sighting, before the ships are again 17 km apart.

OVER→

3. *(a)* A particle is projected from the ground with a velocity of 50.96 m/s at an angle $\tan^{-1}\frac{5}{12}$ to the horizontal. On its upward path it just passes over a wall 14.7 m high. During its flight it also passes over a second wall 18.375 m high.

Show that the second wall must be not less than 23.52 m and not more than 70.56 m from the first wall.

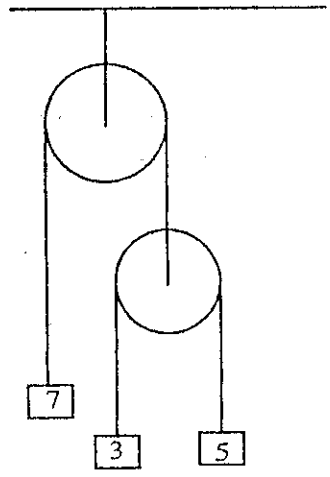
(b) A plane is inclined at an angle of 2β to the vertical. A particle is projected up the plane with initial velocity $u\cos\beta$ at an angle β to the inclined plane. The plane of projection is vertical and contains the line of greatest slope.

Show

(i) that the time of flight of the particle is $\frac{u}{g}$

(ii) that the range of the particle on the plane is $\frac{u^2}{2g}$.

4. A light inextensible string passes over a smooth fixed pulley. It carries at one end a particle of mass 7 kg and at the other end a light, smooth pulley over which passes a light string with particles of mass 3 kg and 5 kg at its ends.



(i) On separate diagrams show the forces acting on each particle and on the movable pulley.

(ii) Find the accelerations of the three particles when the system is released from rest.

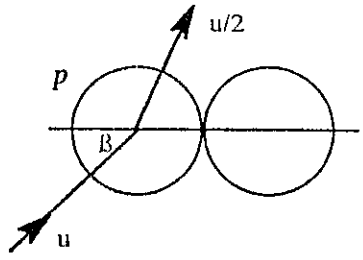
(iii) If the 3 kg mass is replaced by a mass of m kg, find the value of m if this particle does not move when the system is released from rest.

5. *(a)* Two smooth spheres of masses $2m$ and m moving in opposite directions with speeds u and $2u$, respectively, collide directly. If E_1 and E_2 are the sums of the kinetic energies of the two spheres before and after impact respectively, prove that

$$e = \sqrt{\frac{E_2}{E_1}}$$

where e is the coefficient of restitution.

(b) A smooth sphere P, moving with velocity u, impinges on an equal smooth sphere at rest, the direction of u just before impact being inclined at an angle β to the line of centres. If the speed of P after impact is $\frac{u}{2}$ and $\tan\beta = \frac{1}{2}$, show that the coefficient of restitution is also $\frac{1}{2}$.



6. *(a)* A body of mass 10 kg moves with simple harmonic motion. At a displacement of 0.8 m from the centre of oscillation, the velocity and acceleration of the body are 2 m/s and 20 m/s^2 respectively.

Find

(i) the number of oscillations per second

(ii) the amplitude of motion

(iii) the maximum acceleration and hence show that the force to overcome the inertia of the body at the extremity of the oscillation is 223.6N.

(b) A light perfectly elastic string of natural length a and elastic constant k is fastened at one end p to a fixed point of a smooth horizontal table, and a particle of mass m is attached to the other end. The particle is held on the table at a distance $2a$ from p and then released.

Prove

(i) that the particle executes simple harmonic motion while the string is taut

(ii) that the particle reaches p after

$$\left(\frac{\pi}{2} + 1\right)\sqrt{\frac{m}{k}} \text{ seconds.}$$

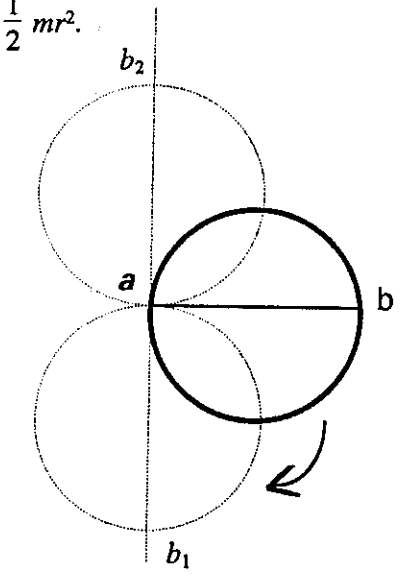
7. *(a)* A rod $[ac]$ of weight W and length 2 m has a particle of weight 2W fixed at a point on the rod 50 cm from a. The rod is kept at rest in a horizontal position by the action of three forces applied at a, b and c where b is the midpoint of $[ac]$. The force at b is four times the force at c.
Calculate the force at a in terms of W.

(b) Two uniform ladders $[ab]$ and $[ac]$, of equal length l and equal weight W, are smoothly jointed at a and stand with b and c in contact with a rough horizontal plane. The coefficient of friction at b and c is μ. If a person of weight W can stand anywhere on the ladders when b and c are a distance $2d$ apart, prove that

$$\mu \geq \frac{2d}{3\sqrt{l^2 - d^2}}.$$

8. *(a)* Prove that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

(b) A uniform circular disc of radius r can move freely about a smooth pivot at a point a on its circumference. When its plane is vertical and the diameter $[ab]$ is horizontal the point b is given a velocity p vertically downwards.



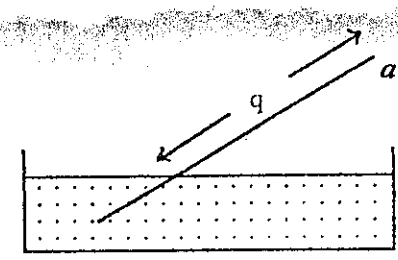
Find

(i) the angular velocity of the disc when b is vertically below a, i.e. at b_1

(ii) the value of p, in terms of r, if b just reaches the point where it is vertically above a, i.e. at b_2.

OVER→

9. *(a)* A uniform rod of length h and relative density s, is pivoted at one end a and is free to move about a horizontal axis. The other end of the rod is immersed in water. A length q of the rod remains above the surface of the water.



Show that $q^2 = h^2 (1 - s)$.

(b) When equal volumes of two substances are mixed the relative density of the mixture is 2.5. When equal weights of the same two substances are mixed the relative density of the mixture is 2.4.

Find the relative densities of the two substances.

10. *(a)* Solve the differential equation

$$\frac{dy}{dx} = 4y \cos x$$

if $y = e^2$ when $x = \frac{\pi}{6}$.

(b) A particle of mass m is projected vertically upwards with a velocity v of $\sqrt{\frac{2g}{k}}$, the air resistance being kv^2 per unit mass. Prove that

(i) the greatest height reached by the particle is $\frac{\ln 3}{2k}$

(ii) the velocity of the particle when passing through the point of projection on the way down is $\sqrt{\frac{2g}{3k}}$.