

# AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1994

## APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 24 JUNE – MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

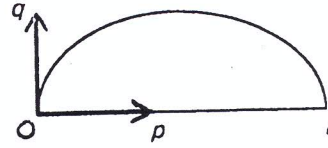
Take the value of  $g$  to be  $9.8 \text{ m/s}^2$ .

**Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.**

---

1. (a) A lift, in a continuous descent, had uniform acceleration of  $0.6 \text{ m/s}^2$  for the first part of its descent and a retardation of  $0.8 \text{ m/s}^2$  for the remainder. The time, from rest to rest, was 14 seconds.
- Draw a time-velocity graph and hence, or otherwise, find the distance descended.
- (b) In a lift, moving upwards with acceleration  $f$ , a spring balance indicates an object to have a weight of 98 N. When the lift is moving downwards with acceleration  $2f$  the weight appears to be 68.6 N.
- Calculate
- (i) the actual weight
  - (ii) the downward acceleration of the lift.
2. A cyclist A is pedalling at 3 m/s due east along a straight road. A second cyclist B is pedalling at 4 m/s due north along another straight road intersecting the first at a junction  $p$ .
- (a) If A is 80 m and B is 40 m from  $p$  at a given moment, calculate
- (i) the velocity of B relative to A.
  - (ii) how far each cyclist is from  $p$  when they are nearest together.
- (b) If when A and B are 80 and 40 m from  $p$ , respectively, then A immediately accelerates at  $0.1 \text{ m/s}^2$  and B decelerates at  $q \text{ m/s}^2$ .
- (i) Find the velocity of B relative to A in terms of time  $t$ .
  - (ii) Determine the value of  $q$  which causes them to arrive at  $p$  together.

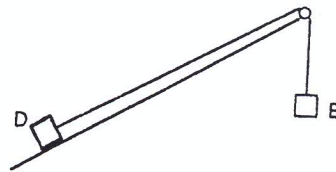
3. (a) A ball is kicked from level ground. The first bounce occurs at the point  $r$ , 45m from the kicking point  $O$  and the greatest height reached was 22.5m. If the horizontal and vertical components of the initial velocity are taken as  $p$  and  $q$  as in the diagram, calculate



- (i) the value of  $p$  and the value of  $q$ .
  - (ii) the farthest distance from  $r$  that a person running at  $7 \text{ m/s}$  can be, so that starting when the ball was kicked, the person can be at  $r$  just as the ball lands.
- (b) A dart-player stood 3 m from a dart-board hanging on a vertical wall. The dart is thrown horizontally from a point 1.8 m above the ground. It strikes the board at a point 1.5 m above the ground. Calculate:
- (i) the initial speed of the dart.
  - (ii) the speed of the dart on striking the board.

4. A particle  $D$ , of mass  $m$ , placed on a rough plane inclined at an angle of  $\tan^{-1} \frac{5}{12}$  to the horizontal, is attached to one end of an inextensible string. The string passes over a small smooth pulley at the top of the plane. An identical particle  $E$  hangs freely from the other end of the string. The particles are released from rest. The coefficient of friction,  $\mu$ , between  $D$  and the plane is  $\frac{1}{3}$ .

- (i) On separate diagrams show the forces acting on each particle and on the pulley.
- (ii) Find the tension in the string.
- (iii) The string broke after two seconds. Find the total distance travelled by  $D$  before coming to rest for the first time.

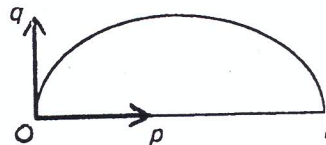


5. A small smooth sphere moves on a smooth horizontal table and strikes an identical sphere lying at rest on the table at a distance of 1 m from a vertical wall, the impact being along the line of centres and perpendicular to the wall. Prove that the next impact between the two spheres will take place at a distance

$$\frac{2e^2}{1+e^2} \text{ metres}$$

from the wall, where  $e$  is the coefficient of restitution for all impacts involved.

3. (a) A ball is kicked from level ground. The first bounce occurs at the point  $r$ , 45m from the kicking point  $O$  and the greatest height reached was 22.5m. If the horizontal and vertical components of the initial velocity are taken as  $p$  and  $q$  as in the diagram, calculate

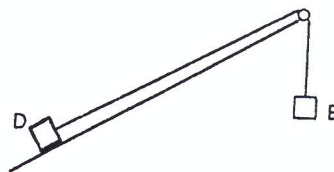


- (i) the value of  $p$  and the value of  $q$ .
  - (ii) the farthest distance from  $r$  that a person running at  $7 \text{ m/s}$  can be, so that starting when the ball was kicked, the person can be at  $r$  just as the ball lands.
- (b) A dart-player stood 3 m from a dart-board hanging on a vertical wall. The dart is thrown horizontally from a point 1.8 m above the ground. It strikes the board at a point 1.5 m above the ground. Calculate:

- (i) the initial speed of the dart.
- (ii) the speed of the dart on striking the board.

4. A particle D, of mass  $m$ , placed on a rough plane inclined at an angle of  $\tan^{-1} \frac{5}{12}$  to the horizontal, is attached to one end of an inextensible string. The string passes over a small smooth pulley at the top of the plane. An identical particle E hangs freely from the other end of the string. The particles are released from rest. The coefficient of friction,  $\mu$ , between D and the plane is  $\frac{1}{3}$ .

- (i) On separate diagrams show the forces acting on each particle and on the pulley.
- (ii) Find the tension in the string.
- (iii) The string broke after two seconds. Find the total distance travelled by D before coming to rest for the first time.



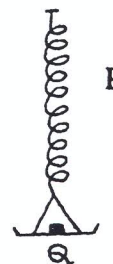
5. A small smooth sphere moves on a smooth horizontal table and strikes an identical sphere lying at rest on the table at a distance of 1 m from a vertical wall, the impact being along the line of centres and perpendicular to the wall. Prove that the next impact between the two spheres will take place at a distance

$$\frac{2e^2}{1+e^2} \text{ metres}$$

from the wall, where  $e$  is the coefficient of restitution for all impacts involved.

6.

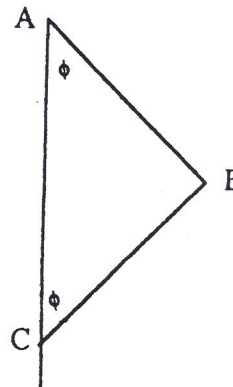
A scale pan is suspended from a fixed point P by a light elastic spring. A particle Q of mass 0.2 kg is attached to the pan with glue. The pan is pulled down from its equilibrium position and set in motion. Given that the motion of Q is simple harmonic, with period  $\frac{\pi}{6}$  seconds and that the maximum and minimum distances of Q below P are 1.5 m and 0.9 m, respectively, calculate



- (i) the maximum speed of Q.
- (ii) the maximum force that the glue has to exert on Q.
- (iii) the length of the spring, when, in the absence of glue, Q would leave the pan.

7.

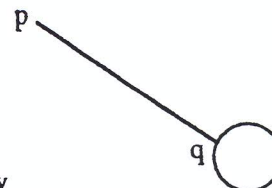
Two uniform rods AB and BC each of length  $l$  and weight  $W$  are smoothly jointed at B. The end A is fixed by a smooth hinge to a rough vertical wall. The system rests in equilibrium in a vertical plane perpendicular to the wall with C in contact with the wall and each rod inclined at an angle  $\phi$  to the vertical.



- (i) Find the frictional force and the normal reaction at C.
- (ii) A force  $W$ , applied vertically downwards at C, is just sufficient to cause slipping. Show that the coefficient of friction between C and the wall is  $\frac{4}{\tan \phi}$ .

8.

A pendulum consists of a rod pq of mass  $m$  and length  $3r$  attached to the rim of a disc of mass  $2m$  and radius  $r$ , as shown. The compound body is set in motion about an axis through p, which is perpendicular to the plane of the rod and the disc.



- (i) Find the period of small oscillations.
- (ii) If the pendulum is released from rest when pq makes an angle  $\theta$  with the downward vertical, show that the angular speed acquired is given, by  $\frac{19g}{36r} (\cos \alpha - \cos \theta)$  when the angle made by pq with the downward vertical is  $\alpha$ .

9. (a) A submarine of mass 200 tonnes is completely immersed in sea-water of density  $1030 \text{ kg/m}^3$ .
- (i) What is the volume of water displaced ?
- (ii) What weight of water must be pumped from the submarine's ballast tanks if it is to just float in fresh water ?
- (b) A liquid of density  $p$  rests on another liquid of density  $q$  without mixing. A solid of density  $d$  floats with its surface totally covered by liquid and with part of its volume immersed in the lower liquid. Show that the fraction of the volume of the solid immersed in the lower liquid is

$$\frac{d-p}{q-p}$$

10. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{y - xy}{1 + x}$$

if  $y = 1$  when  $x = 0$ .

- (b) A car of mass 1000 kg moves with velocity  $v$  m/s along a horizontal road against a constant resistance of 1500 N. The engine is working at a constant rate of 75 kW.
- (i) Show that the acceleration of the car is  $\frac{150 - 3v}{2v} \text{ m/s}^2$ .
- (ii) Calculate, correct to two decimal places, the time taken by the car to increase its speed from 0 m/s to 25 m/s.

# AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1993

## APPLIED MATHEMATICS - HIGHER LEVEL

FRIDAY, 25 JUNE - MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of  $g$  to be  $9.8 \text{ m/s}^2$ .

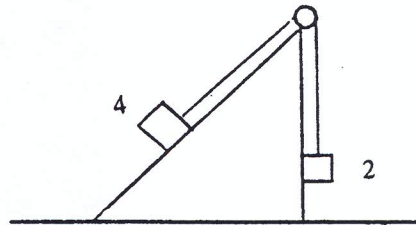
Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

---

1. (a) A particle moving in a straight line travels 30 m, 54 m and 51 m in successive intervals of 4, 3 and 2 seconds.
- (i) Verify that the particle is moving with uniform acceleration.
- (ii) Draw an accurate speed-time graph of the motion.
- (b) A particle P is projected vertically upwards from the ground with an initial velocity of 47 m/s. Two seconds later another particle Q is projected vertically upwards from the same point with initial velocity 64.6 m/s. Calculate
- (i) how long Q is in motion before it collides with P.
- (ii) the height at which the collision occurs.
2. (a) A girl travelling south at 11 m/s finds that the wind appears to blow from the East but on doubling her speed it appears to come from the South-east.
- Calculate the magnitude and direction of the wind's velocity.
- (b) A boy who can swim at  $5/9$  m/s wishes to cross a river 50 m wide, flowing at  $5/6$  m/s, as quickly as possible. Calculate
- (i) the direction he should take.
- (ii) the time he takes to cross.
- (iii) how far downstream from his starting point he goes.

3. (a) A particle is projected on a horizontal plane with initial velocity  $u$  at an angle  $\beta$  to the horizontal. If the range of the projectile is three times the greatest height, prove that  $\tan \beta = 4/3$ .
- (b) A particle is projected up an inclined plane with initial speed  $u$ . The line of projection makes an angle of  $30^\circ$  with the plane and the plane is inclined  $30^\circ$  to the horizontal. (The plane of projection is vertical and contains the line of greatest slope.) The particle strikes the plane at an angle  $\theta$ ,  $\theta < 90^\circ$  (i.e.  $\theta$  is the landing angle).
- (i) Express the velocity and displacement of the particle after  $t$  seconds in terms of unit vectors  $\vec{i}$  and  $\vec{j}$  along and perpendicular to the plane, respectively.
- (ii) calculate  $\theta$ .

4. A smooth wedge, of mass 10 kg and slope  $\tan^{-1} 3/4$ , is placed on a smooth horizontal surface. A particle of mass 4 kg is placed on one face and it is connected by a light inextensible string which passes over a light frictionless pulley to a second particle of mass 2 kg which hangs vertically and touches the side of the wedge.



- (i) Show, on separate diagrams, the forces acting on the wedge, the 4 kg mass and the 2 kg mass.
- (ii) Prove that the acceleration of the wedge is  $g/67$ .

5. State the laws governing the oblique collision between two smooth elastic spheres.

A smooth sphere A, of mass  $m$ , moving with speed  $u$ , collides with a smooth sphere B, of mass  $m$ , which is at rest. The direction of motion of A before and after impact makes angles  $\cos^{-1} \frac{5}{\sqrt{35}}$  and  $\cos^{-1} \frac{3}{7}$  respectively with the line of centres. The coefficient of restitution between A and B is  $2/5$ . Show that after impact, A and B have the same speed.

Calculate the loss of kinetic energy due to the impact.

6. (a) A particle of mass  $m$  moves with simple harmonic motion under the action of a variable force. If the maximum value of the force is  $\frac{7m}{16}$  and the amplitude of the motion is 4 m calculate

- (i) the period of the oscillation.  
(ii) the speed of the particle at a time  $\frac{2\pi}{\sqrt{7}}$  seconds after passing through the centre of oscillation.

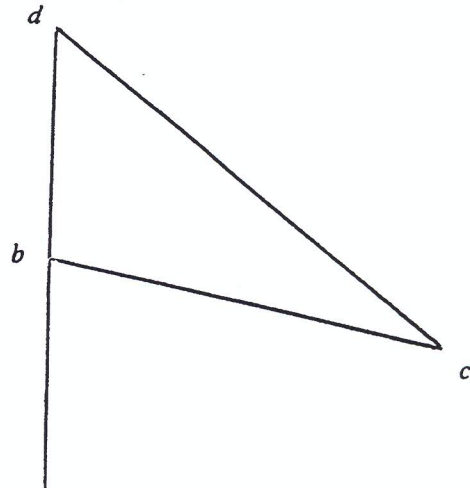
- (b) A light elastic string, of elastic constant  $\frac{48mg}{l}$  and natural length  $l$  has one end attached to a fixed point. Two particles of masses  $3m$  and  $2m$  are attached to the other end and the system hangs in equilibrium. If the  $2m$  mass falls off

- (i) prove that the  $3m$  mass will move with simple harmonic motion of period

$$\frac{\pi}{2} \sqrt{\frac{l}{g}}$$

- (ii) find the amplitude of the motion.

7. (a) A uniform rod  $bc$ , of weight  $W$  and length  $2l$ , rests in equilibrium with  $b$  in contact with a rough vertical wall. One end of a light inextensible string is fixed to a point  $d$  on the wall vertically above  $b$ , the other end is attached to  $c$ .



- (i) If  $\angle bdc = 30^\circ$  and  $\angle dbc = 120^\circ$ , prove that  $\mu \geq 1/\sqrt{3}$  where  $\mu$  is the coefficient of friction between the rod and the wall.

- (ii) If  $\mu = \frac{1}{\sqrt{3}}$  and the string is shortened so that the  $\angle bdc = \theta$ , where  $\theta > 30^\circ$ , prove that the rod slips down the wall.

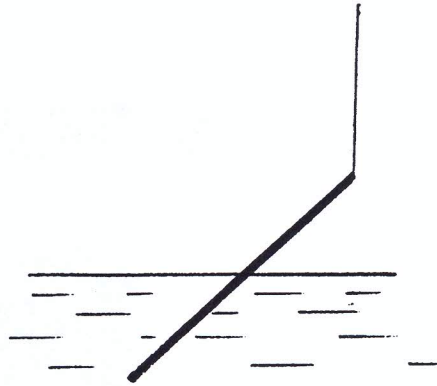
8. (a) Prove that the moment of inertia of a uniform rod of mass  $m$  and length  $2l$ , about an axis through its centre of mass perpendicular to the rod is  $\frac{1}{3}m^2$ .

- (b) A uniform rod of mass  $m$  and of length 1.2 m swings in a vertical plane about a horizontal axis through the rod at a distance of 0.4 m from its upper end.

- (i) If  $v$  m/s is the velocity of the lower end when the rod is vertical, prove that the rod will make a complete revolution if  $v \geq 5.6$  m/s.  
(ii) If a mass  $m$  is attached to each end of the rod and the compound body is set in motion calculate the period of small oscillations correct to two decimal places.



9. (a) A solid, uniform cylinder 20 cm long floats in water with its axis vertical and 17 cm of its length immersed. Oil of relative density 0.8 is poured on to the water until the top of the cylinder is in the oil surface. What is the depth of the layer of oil?
- (b) A thin uniform rod, floats motionless in water in an inclined position. The upper end of the rod is supported by a string.
- (i) Prove that the string is vertical.
- (ii) If the relative density of the rod is 0.64, calculate the length of the immersed part of the rod.



10. (a) If  $(x^2 + 2) \frac{dy}{dx} = x(y + 1)$  and  $y = 2$  when  $x = 1$ , find the value of  $y$  when  $x = 2$ .
- (b) A particle starts with a speed of 20 m/s and moves in a straight line. The particle is subjected to a resistance which produces a retardation which is initially  $8 \text{ m/s}^2$  and which increases uniformly with the distance moved, having a value of  $9 \text{ m/s}^2$  when the particle has moved a distance 5 m.
- If  $v \text{ m/s}$  is the speed of the particle when it has moved a distance  $x \text{ m}$
- (i) prove that, while the particle is in motion,
- $$v \frac{dv}{dx} = - \left( 8 + \frac{x}{5} \right)$$
- (ii) calculate the distance moved by the particle in coming to rest.

# AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1992

## APPLIED MATHEMATICS - HIGHER LEVEL

FRIDAY, 26 JUNE - MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of  $g$  to be  $9.8 \text{ m/s}^2$ .

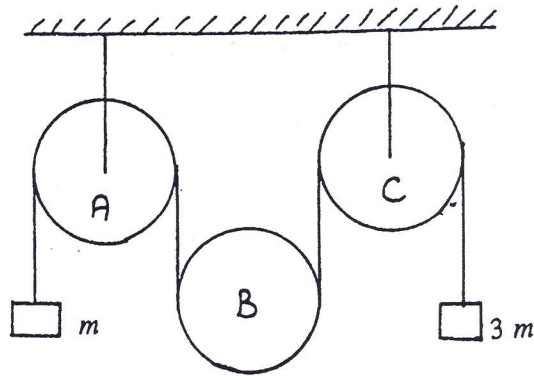
Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

---

1. (a) A balloon ascends vertically at a uniform speed.  
7.2 seconds after it leaves the ground a particle is let fall from the balloon.  
The particle takes 9 seconds to reach the ground.  
Calculate the height from which the particle was dropped.
- (b) Two particles P and Q are moving in the same direction along parallel straight lines with accelerations  $5 \text{ m/s}^2$  and  $4 \text{ m/s}^2$ , respectively. At a certain instant P has a velocity  $1 \text{ m/s}$  and Q is  $25.5 \text{ m}$  behind P moving with velocity  $11 \text{ m/s}$ .
- (i) Prove that Q will overtake P and that P will in turn overtake Q.
- (ii) When Q is in front of P find the greatest distance between the particles.
2. An aeroplane having a speed of  $500 \text{ km/h}$  in still air travels  $1500 \text{ km}$  due North when the wind is blowing from  $60^\circ$  East of North at  $90 \text{ km/h}$ , and then returns to the starting point along the same path. Calculate
- (i) the directions in which the aeroplane must travel on the outward and return journeys.
- (ii) the total time taken
- (iii) the total time taken if there was no wind blowing.
3. A particle is projected with velocity  $6\sqrt{g} \text{ m/s}$ , at an angle  $\alpha$  to the horizontal, from a point  $18 \text{ m}$  in front of a vertical wall  $5.5 \text{ m}$  high.
- (i) Calculate the two possible values of  $\alpha$  which will enable the particle to just clear the wall.
- (ii) Show that the value of  $\alpha$  is  $\tan^{-1} 2$  for maximum clearance height.

4.

A light inextensible string passes over a fixed pulley A, under a movable pulley B, of mass  $M$ , and then over a second fixed pulley C. A mass  $m$  is attached to one end of the string and a mass  $3m$  is attached to the other end. If the system is released from rest



(i) Show in a diagram the forces acting on each of the three masses.

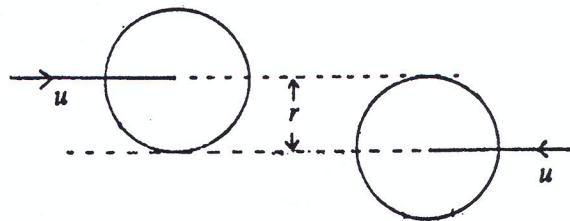
(ii) Prove that the tension,  $T$ , of the string is given by the equation

$$T \left( \frac{1}{M} + \frac{1}{3m} \right) = g.$$

(iii) Show that if  $M = 3m$  then pulley B will remain at rest while the two masses are in motion.

5.

Two equal smooth spheres, of radius  $r$ , move horizontally in opposite directions with speed  $u$ . Their centres lie on two parallel lines a distance  $r$  apart. The coefficient of restitution is  $\frac{1}{3}$ .



(i) Prove that at the moment of impact the line of centres makes an angle of  $30^\circ$  with the previous direction of motion.

(ii) Find the velocity of each sphere after impact.

(iii) What fraction of the kinetic energy is lost as a result of the collision.

6. (a) If the displacement of a moving particle at any time  $t$  is given by the equation

$$x = 5 \cos \omega t + 12 \sin \omega t$$

- (i) show that the motion is a simple harmonic motion.  
(ii) calculate the amplitude of the motion.

- (b) A particle of mass 2 kg is attached to one end of a light elastic string of natural length 1 m and elastic constant 14 N/m. The other end of the string is fixed to a point A on a smooth horizontal table. The particle is pulled across the table and released from rest at a point C which is a distance 1.5 m from A.  
If B is a point on AC such that  $|AB| = 1\text{m}$ ,

- (i) prove that the particle performs simple harmonic motion when travelling from C to B.  
(ii) calculate the time taken to travel from C to B.  
(iii) prove that the particle then travels for  $\frac{4}{\sqrt{7}}$  s with constant speed.

7. (a) A body of weight 14 N is kept at rest on a smooth plane of inclination  $\alpha$  by a horizontal force of 7N together with a force of 7N acting up along the line of greatest slope of the plane.

Show that  $\cos \alpha = \frac{3}{5}$ .

- (b) Two equal uniform rods  $pq$  and  $qr$ , each of weight  $W$ , are freely jointed at  $q$ . The rods are in a vertical plane and the ends  $p$  and  $r$  rest on uniformly rough horizontal ground.

- (i) If the rods are on the point of slipping, prove

$$\mu = \frac{1}{2 \tan \alpha}$$

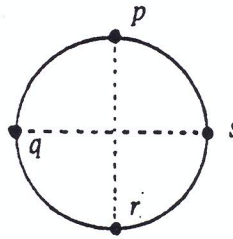
where  $\mu$  is the coefficient of friction and  $\alpha$  is the inclination of each rod to the horizontal.

- (ii) If  $\mu > \frac{1}{2 \tan \alpha}$  and  $\mu \tan \alpha < 1$ , find the maximum weight which can be placed at  $q$  without slippage taking place.

8. (a) Prove that the moment of inertia of a uniform circular disc, of mass  $m$ , and radius  $r$ , about an axis through its centre perpendicular to its plane is  $\frac{1}{2} mr^2$

(b) Particles, each of mass  $m$ , are fixed at  $q$ ,  $r$  and  $s$  which are on the circumference of a uniform circular disc of mass  $8m$  and radius  $r$ .

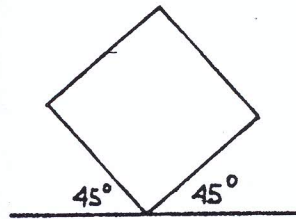
$p$ ,  $q$ ,  $r$ , and  $s$  are the extremities of two perpendicular diameters. The system can turn freely in a vertical plane about  $p$ . Calculate the period of small oscillations.



9. (a) State the Principle of Archimedes.

A buoy in the form of a hollow spherical shell of external radius 0.8 m and internal radius  $r$  floats in water with half of its volume immersed. If the relative density of the material of the shell is 5.12, calculate the thickness of the shell.

(b) A closed tank in the shape of a cube of side  $\sqrt{2}$  is half full with water and half with oil of relative density 1.2. The tank is placed on a horizontal table and is then tilted about one edge until the faces about this edge are inclined at  $45^\circ$  to the horizontal.



Find the thrust on one of the vertical faces of the tank.

(i) if the oil and water are not mixed.

(ii) if the oil and water are mixed and there is no reduction in the volume as a result of mixing.

10. (a) If

$$\frac{dy}{dx} = 2 \sin 2x + \cos 4x$$

and if  $y = 1$  when  $x = \frac{\pi}{4}$ , find the value of  $y$  when  $x = \frac{\pi}{2}$ .

(b) A particle experiences a retardation of  $kv \text{ m/s}^2$  when its velocity is  $v \text{ m/s}$ . Its velocity is reduced from its initial value of 210 m/s to 70 m/s in 0.5 s and it travels a distance  $x \text{ m}$  in this time.

(i) Find the value of  $k$  and deduce an expression for the velocity at any time  $t$ .

(ii) Calculate the value of  $x$ .

LEAVING CERTIFICATE EXAMINATION, 1991

APPLIED MATHEMATICS - HIGHER LEVEL

FRIDAY, 21 JUNE - MORNING, 9.30 - 12.00

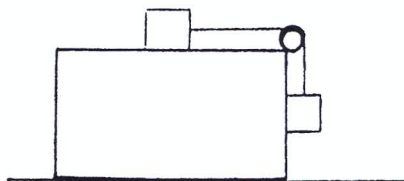
Six questions to be answered. All questions carry equal marks.  
Mathematics Tables may be obtained from the Superintendent.  
Take the value of  $g$  to be  $9.8 \text{ m/s}^2$ .  
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) A particle starts from rest at a point  $p$  and accelerates at  $2 \text{ m/s}^2$  until it reaches a speed  $v \text{ m/s}$ . It travels at this speed for 1 minute before decelerating at  $1 \text{ m/s}^2$  to rest at  $q$ . The total time for the journey is 2 minutes.
- (i) Calculate the distance  $pq$ .
- (ii) If a second particle starts from  $p$  at time  $t = 0$  and moves along  $pq$  with speed  $(2t + 50) \text{ m/s}$ , find the time taken to reach  $q$ .
- (b) A particle  $P$  is projected vertically upwards with an initial velocity  $u$  and two seconds later a second particle  $Q$  is projected vertically upwards from the same point with initial velocity  $1.5u$ . Calculate, in terms of  $u$ , how long  $Q$  is in motion before it collides with  $P$  and prove that  $|u| > 9.8$ .
2. A runner observes that when her velocity is  $u\vec{i}$  the wind appears to come from the direction  $2\vec{i} + 3\vec{j}$ , but when her velocity is  $u\vec{j}$  the wind appears to come from the direction  $-2\vec{i} + 3\vec{j}$ .
- (i) Prove that the true velocity of the wind is

$$\frac{5u}{6} \vec{i} - \frac{u}{4} \vec{j}$$

- (ii) Find the speed and direction of motion of the runner when the wind velocity appears to be  $u\vec{i}$ .
3. A particle is projected, with speed  $u$ , down a plane which is inclined at an angle of  $30^\circ$  to the horizontal. The plane of projection is vertical and contains the line of greatest slope. The coefficient of restitution between the particle and the plane is  $e$ . The direction of projection makes an angle of  $60^\circ$  with the inclined plane.
- (i) Find the range to the first hop.
- (ii) For what value of  $e$  is the range for the second hop double the range for the first hop?
4. A rectangular block moves across a stationary horizontal surface with acceleration  $g/3$ . A particle of mass  $m$ , on the block, is connected by a string which passes over a light, smooth, fixed pulley to a second particle of mass  $m$  which presses against the block (see diagram).

- (i) If contact between the particles and the block, is smooth, find the magnitude and direction of the resultant forces acting on the particles.
- (ii) If contact between the particles and the block, is rough, for what same value of the coefficient of friction, will the particles remain at rest relative to the block ?



5. (a) A sphere of mass  $4m$  travelling with speed  $u$ , strikes directly a stationary sphere of mass  $2m$ . If the coefficient of restitution is  $e$ , prove that the energy lost in the collision is

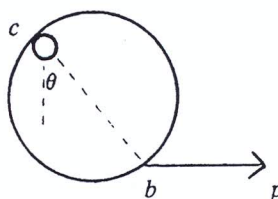
$$\frac{2mu^2}{3} (1 - e^2)$$

- (b) A smooth sphere  $P$ , moving with velocity  $4\vec{i} + 5\vec{j}$  m/s collides with an identical sphere  $Q$  moving with velocity  $2\vec{i} + 3\vec{j}$  m/s where  $\vec{i}$  is a unit vector along the line of centres at the moment of impact. If  $e$  is the coefficient of restitution
- (i) find the velocity of each sphere after impact.
- (ii) calculate the value of  $e$  if  $P$  is deflected through an angle  $\tan^{-1}(\frac{1}{4})$  as a result of the collision.

6. (a) A light elastic string of natural length  $1.2$  m is found to extend by  $20$  cm when a small mass is gently attached to one end, the other end being fixed. The mass is now made to describe a horizontal circle with angular velocity  $w$  rad/s. Find an expression for the extension of the string in terms of  $w$ .
- (b) A particle of mass  $3$  kg is suspended by a light string which is found to extend by  $20$  cm when the particle is at rest. If the upper end of the string is held firm and the particle is pulled down slightly and then released, find the period of the resultant motion.

7. A uniform circular hoop of weight  $W$  hangs over a rough horizontal peg  $c$ . A horizontal force  $p$  is applied at a point  $b$  where  $cb$  is a diameter of the hoop. When  $cb$  is inclined at an angle  $\theta$  to the downward vertical the system is in equilibrium and the hoop has not slipped.

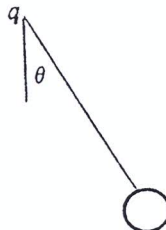
- (i) Find  $p$  in terms of  $W$  and  $\theta$ .
- (ii) Show that the ratio of the frictional force to the normal reaction at the peg is  $\frac{\tan \theta}{2 + \tan^2 \theta}$



- (iii) Show that, when the coefficient of friction is  $1/2$ , the hoop never slips.

8. (a) Prove that the moment of inertia of a uniform rod of mass  $m$  and length  $2l$ , about an axis through its centre of mass perpendicular to the rod is  $\frac{1}{3} ml^2$

- (b) A uniform rod of mass  $m$  and length  $88$  cm has a uniform disc of mass  $m$  and radius  $12$  cm attached to one end. The rod and disc are in the same plane and the rod is collinear with a diameter of the disc (see diagram).



If the compound body is set in motion about an axis through  $g$  which is perpendicular to the plane of the rod and disc,

- (i) find the period of small oscillations correct to two decimal places.
- (ii) find the length of the equivalent simple pendulum.

9. State the Principle of Archimedes.

A uniform rod, of length  $p$  and relative density  $s$  is free to turn about its lower end, which is fixed at a depth  $h$  in a liquid of relative density  $q$ .

- (i) Prove that the rod can float in an inclined position if
$$p^2 s > h^2 q$$
- (ii) Calculate the value of  $s$  if the rod floats with three-quarters of its length immersed in water.

10. (a) Solve the differential equation

$$x \frac{dy}{dx} = \frac{1}{y} + y$$

if  $y = 1$  when  $x = 1$ .

- (b) A particle is projected in a straight line from a fixed point with velocity  $u$  at time  $t = 0$ . It is opposed by a resistance  $kv^n$  per unit mass. If  $s$  is the displacement at time  $t$  prove that when  $v = 0$

$$s = \frac{u^2 - n}{(2 - n)k} \quad \text{if } n < 1.$$